Quantum vacuum and electromagnetism

L. L. Sánchez-Soto

G. Leuchs
Maxwell's lucky strike

FitzGerald-Lorentz invariance ✓

field modes in quantum optics ✓

connection to high energy physics ?
Beyond the electric field strength, the vacuum is predicted to be non-linear.

\[ E_S \simeq \frac{m^2 c^3}{\epsilon_0 \hbar} \simeq 5 \times 10^9 \text{W/cm}^2 \]

Maxwell Equations

0 \quad E_S \quad \text{electric field}

Linear \quad Non-linear

Bohr, Sauter, Heisenberg & Euler, Schwinger ...

If – over a distance of a Compton wavelength – the virtual particles are accelerated to their rest mass energy, they become real pair production!

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \simeq mc \Rightarrow \Delta x \simeq \frac{\hbar}{mc} = \lambda_{\text{Compton}}$$

Bohr (before 1931), Sauter 1931, Schwinger 1951
A sum rule for charged elementary particles

Gerd Leuchs\textsuperscript{1,2} and Luis L. Sánchez-Soto\textsuperscript{1,2,3,a}

\textsuperscript{1} Max-Planck-Institut für die Physik des Lichts, Günther-Scharowsky-Straße 1, Bau 24, 91058 Erlangen, Germany
\textsuperscript{2} Department für Physik, Universität Erlangen-Nürnberg, Staudtstraße 7, 91058 Erlangen, Germany
\textsuperscript{3} Departamento de Óptica, Facultad de Física, Universidad Complutense, 28040 Madrid, Spain

Received 17 September 2012 / Received in final form 15 January 2013
Published online 21 March 2013 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2013
The problem (I)


We shall treat the polarization of the vacuum in detail in § 32, but the qualitative consequences are easily seen: A constant field merely leads to a constant polarization. The polarizability is field independent and thus all charges are changed by a universal constant factor (the ‘dielectric constant’ of the vacuum). Since there is no way of experimenting in an ‘ideal vacuum’ where this polarizability is absent, this effect is unobservable in principle (although the polarizability turns out to be infinite owing to the infinite number of pairs which contribute).

An inhomogeneous field creates, in addition, a non-uniform polarization whose effects are observable (and finite). For example, an additional light wave will be scattered by the polarization dipoles, i.e. in the Coulomb field. Similarly, two light waves will be scattered by each other (§ 32).
V. S. Weisskopf: Kongelige Danske Videnskabernes Selskab, Mathematisk-fysiske Meddelelser XIV, No.6, (1936)

Weisskopf’s three problematic quantities

1.- ...
2.- ...
3.- A spatially and temporally constant and field independent electric and magnetic polarizability of the vacuum

These quantities relate to the field free vacuum. It can be taken as self-evident that they can have no physical meaning.
Probing the quantum vacuum

- Warning

\[ c_{\text{rel}} = \sqrt{\frac{E}{m}} \]

are not necessarily equivalent!


- We think of the vacuum as a dielectric and diamagnetic medium

\[ D = \varepsilon_0 E + P \equiv P_0 + P \]

\[ H = \frac{1}{\mu_0} B - M \equiv M_0 - M \]

derive \( \varepsilon_0, \mu_0 \) from properties of the vacuum
Dielectrics: the naive picture (I)

\[
p = 0 \quad \text{without external field}
\]

\[
p = q \cdot r \quad \text{with external field}
\]

polarization = \frac{\text{dipole moment}}{\text{volume of dipole}}
Dielectrics: the naive picture (II)
The ether

• Absolute reference system

• Incompressible

• Huge stiffness

• Not opposing to the motion of bodily matter

• Experiments cannot detect the relative motion respect to the ether!

The Persistence of Ether

Frank Wilczek

Quite undeservedly, the ether has acquired a bad name. There is a myth, repeated in many popular presentations and textbooks, that Albert Einstein swept it into the dustbin of history. The real story is more complicated and interesting. I argue here that the truth is more nearly the opposite: Einstein first purified, and then enthroned, the ether concept. As the 20th century has progressed, its role in fundamental physics has only expanded. At present, renamed and thinly disguised, it dominates the accepted laws of physics. And yet, there is serious reason to suspect it may not be the last word.
The quantum vacuum
\[ \langle d \rangle = e \langle x \rangle = \frac{e^2}{m\omega_0^2} \zeta E = \frac{e^2 \hbar^2 \zeta}{2m^3 c_{\text{rel}}^4} E \]

Transient creation of the dipole
The quantum vacuum (III)

\[ V = \eta \left( \frac{\hbar}{mc_{rel}} \right)^3 \]

\text{~ Compton wavelength} \, \frac{\hbar}{mc_{rel}}
Mass drops out!

\[ P_0 = \frac{d}{V} = \frac{e^2 \zeta}{2c_{\text{rel}} \hbar \eta} E \]

What are the different types of elementary particles that contribute?

\[ \begin{array}{cccc}
e & \mu & \tau \\
u & d & c & s & t & b \end{array} \times 3 \]

\[ W^{\pm/} \ldots \]

\[ \varepsilon_0 = \frac{\zeta}{2c_{\text{rel}} \hbar \eta} \left( \sum_j e.p. q_j^2 \right) \]
The magnetic response

\[ d_{\text{magn}} = 2iA = 2(e\nu)(\pi \rho^2) = \frac{e^2}{m} \rho^2 B \]

\[ \rho^2 = \xi \left( \frac{\hbar}{m_j c_{\text{rel}}} \right)^2 \]

\[ M_0 = \frac{\xi c_{\text{rel}} e^2}{\eta \hbar} \]

\[ \frac{1}{\mu_0} = \frac{\xi c_{\text{rel}}}{\eta \hbar} \left( e \cdot p \sum_j q_j^2 \right) \]
Vacuum impedance determines the power radiated by a dipole

\[ P = e^2 \omega^2 \left( \frac{d}{\lambda} \right)^2 \frac{\pi}{3} Z_0 \]

\[ Z_0 = 376.7 \text{ Ohms} \]
Vacuum impedance

\[ c_{\text{light}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad \quad \quad c_{\text{light}} = c_{\text{rel}} \sqrt{\frac{2\xi}{\zeta}} \]

\[ \frac{\xi}{\zeta} = 2 \]

\[ Z_0 = \frac{2\eta \hbar}{\zeta} \left( \sum_j \frac{q_j^2}{e^2} \right)^{-1} = 8218[\Omega] \frac{\eta}{\zeta} \left( \sum_j \frac{q_j^2}{e^2} \right)^{-1} \]

\[ \sum_j \frac{q_j^2}{e^2} = \frac{2\hbar}{e^2 Z_0 \zeta} = 21.82 \frac{\eta}{\zeta} \]

\[ \eta \approx 0.48 - 2.06 \]

QVG 2013. Toulouse. 6 November 2013
**The running fine structure constant**

\[ \alpha^{-1} = \frac{e^2}{4\pi\epsilon_0} \]

\[ \alpha_0^{-1} = 137.04 = \text{constant} \sum_j \frac{q_j^2}{e^2} \]

\[ \alpha_{58\text{ GeV}}^{-1} = 128.5 \pm 2.5 \]

\[ = \text{constant} \sum_j \frac{q_j^2}{e^2}, \text{ e. p.} > 58 \text{ GeV} \]

---

**Graph:**
- **\( \alpha^{-1}(Q^2) \)**
- **\( \langle Q_{\gamma_1} Q_{\gamma_2} \rangle^{1/2} \)**
- **\( \alpha_{\text{QED}}^{-1}(0) \)**
- **This experiment**
- **TOPAZ hadronic data**

**Equations:**
- \[ \alpha = \frac{e^2}{4\pi\epsilon_0} \]
- \[ \alpha_0^{-1} = 137.04 = \text{constant} \sum_j \frac{q_j^2}{e^2} \]
- \[ \alpha_{58\text{ GeV}}^{-1} = 128.5 \pm 2.5 \]
- \[ = \text{constant} \sum_j \frac{q_j^2}{e^2}, \text{ e. p.} > 58 \text{ GeV} \]
The running fine structure constant

\[
\frac{\alpha_0^{-1}}{\alpha_{58 \text{ GeV}}^{-1}} = \frac{\sum_{\text{all}}}{\sum_{\text{>58 GeV}}} = \frac{137.04}{128.05 \pm 2.5}
\]

\[
\sum_{\text{all}} = 104 \left\{ \begin{array}{c} +43 \\ -24 \end{array} \right\}
\]
• Our simple model predicts a finite number of charged elementary particles and that it relates this number to low-energy properties of the electromagnetic field.

• The value predicted by the model is determined by the relative properties of the electric and magnetic interaction of light with the quantum vacuum and is independent of the number of elementary particles.

• We have shown an intimate relationship between the properties of the quantum vacuum and the constants in Maxwell's equations.
"It is deplorable that fewer and fewer students nowadays study Heitler's classical treatise on the quantum theory of radiation. As a result, we see a number of sophisticated, yet uneducated, theoreticians who are conversant in LSZ formalism of Heisenberg field operators but do not know why an excited atom radiates"