Black Holes in Loop Quantum Gravity

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Quantizing Gravity : Why ?

Two (amazingly efficient) fundamental theories

- ▷ Small scales (up to ...) : Quantum Physics and QFT
- ▷ Large scales (up to ...) : General Relativity

When General Relativity meets Quantum Physics

- ▷ Origin of the Universe : below the Planck length $\ell_P = \sqrt{\frac{hG}{c^3}}$

$$r_S = \frac{2Gm}{c^2} \sim \frac{h}{mc} = \lambda_C$$

- ▷ Black Holes : Bekenstein-Hawking thermodynamics $S = a_H/4\ell_P^2$
- ▷ Problem of Singularities in General Relativity : Penrose

Problems in Quantizing Gravity

- ▷ Perturbative quantization : non-renormalizable theory
- ▷ Hamiltonian quantization : technically too involved
- ▷ What is the meanning of quantizing space-time ?

Quantizing Gravity : How ?

(Inequivalent) Attempts to quantize gravity

▷ String theory : General Relativity should be modified at high energies but quantization rules remain unchanged

▷ Loop Quantum Gravity : GR is fundamental but the quantization process should be modified/adapted

▷ Others : Dynamical triangulations, Non-Commutative Geometry ...

Loop Quantum Gravity : a very powerful approach

▷ The Space is fundamentally discrete

▷ The fundamental length is a UV cutt-off : consequences in LQC

▷ Discreetness provides a statistical explanation of BH thermodynamics

Even though the theory is not yet complete

▷ The kinematics is totally well understood : states and observables

▷ But the quantum dynamics is not fully under control : Spin-Foams

▷ Many open questions concerning the classical limit

Black Holes to probe quantum gravity at the Planck length

Black Hole thermodynamics : the classical viewpoint

- ▷ Quantum Field Theory in a classical curved background
- ▷ Bekenstein-Hawking entropy for any Black Holes : $S = a_H/4\ell_P^2$
- ▷ Where a_H is the horizon area : information is contained in the area
- ▷ Thermal radiation of particles at Hawking temperature $T = \kappa/2\pi$

Quantum Black Holes : old results

- ▷ Precise description of the BH micro states : partition function $\mathcal{Z}$
- ▷ The micro canonical entropy $S = log(\mathcal{Z})$
- ▷ It reproduces the classical result at the semi-classical limit

Complex Quantum Black Holes : new results

- ▷ Effective description for an observer close to a Quantum BH
- ▷ Particles thermalized at temperature T : graviton ?
- ▷ The spectrum : $|j, m\rangle$ where $m = -j, -j+1, \cdots$ and $E_m = mE_0$
- ▷ The BH entropy is the entropy of the vacuum
- ▷ The BH temperature is obtained from the excited states

Overview

1. Loop Quantum Gravity in a nut shell

- *Why standard quantization schemes fail ?*
- *From Ashtekar gravity...*
- *... To kinematical quantum states*
- *Physical interpretation : discrete geometry*

2. Black Holes in LQG: a quick review

- *Heuristic picture : the Rovelli model*
- *Relation to Chern-Simons theory*

3. New results: vacuum, temperature and gravitons

- *Going to complex variables*
- *The new partition function*
- *Vacuum and entropy ; Excited states and temperature*

Why standard quantization schemes fail ?

Lagrangian formulation : M is the 4D space-time

- ▷ Einstein-Hilbert action : functional of the metric g

$$S_{EH}[g] = \int d^4x \sqrt{|g|} R$$

Hamiltonian formulation : $M = \Sigma \times \mathbb{R}$ ('61)

- ▷ ADM variables : $ds^2 = N^2 dt^2 - (N^a dt + h_{ab} dx^b)(N^a dt + h_{ac} dx^c)$
- ▷ ADM action : (h, π) canonical variables

$$S_{ADM}[h, \pi; N, N^a] = \int dt \int d^3x (\dot{h}\pi + N^a H_a[h, \pi] + NH[h, \pi])$$

- ▷ Constraints $H = 0 = H_a$ generate the diffeomorphisms

What about the quantization ?

- ▷ Path integral : non renormalizable theory
- ▷ Hamiltonian : too complicated constraints !

From Ashtekar gravity...

Starting point : first order formulation of gravity

- A tetrad e_μ^I (4×4 matrix) such that $g_{\mu\nu} = e_\mu^I e_\nu^J \eta_{IJ}$
- a $so(3,1)$ spin-connection ω_μ^{IJ} related to Levi-Civitta connection

The Ashtekar variables ('86)

- New complex variables : $E^a = \epsilon^{abc} e_b \times e_c$ and $A_a^i = \omega_a^i + \gamma \omega_a^{0i}$
- Pair of canonical variables :

$$\{A_a^i(x), E_j^b(y)\} = (8\pi\gamma G)\delta_a^b \delta_j^i \delta^3(x,y)$$

- Where $\gamma = \pm i$: Complex (or non-compact) symmetry group
- The constraints become polynomials in E and A
- But... No one knows how to deal with complex variables

Immirzi-Barbero parameter γ

- One considers γ real : canonical transformation
- Interpret as a Wick rotation : gauge group becomes compact $SU(2)$

... To kinematical Quantum States

Schrodinger like quantization

▷ States are functionals $\Psi(A)$ of the connection A

$$\hat{E} \triangleright \Psi(A) = i\gamma\ell_P \frac{\delta\Psi}{\delta A} \quad \text{and} \quad \hat{A} \triangleright \Psi(A) = A\Psi(A)\,.$$

▷ But no measure and no scalar product exists, then no predictions

Polymer quantization

▷ States have support on the one dimensional lines of a graph Γ

▷ Fundamental variables form the holonomy-flux algebra associated to edges e of Γ and surfaces S dual to Γ

$$A(e) = P\exp(\int_e A) \quad \text{and} \quad E_f(S) = \int_S \mathrm{Tr}(f \star E)\,.$$

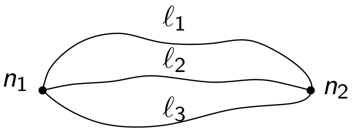
▷ Cylindrical functions : $f \in \mathrm{Cyl}(\Gamma)$ is a function of $A(e) \in SU(2)$

▷ $E_f(S)$ acts as a vector field on f if $S \cap \gamma \neq 0$.

Physical interpretation (Rovelli - Smolin)

Kinematical states : basis of spin-networks

▷ They are generalizations of Wilson loops with nodes

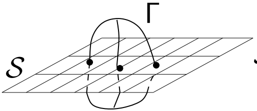


ℓ_i are oriented links

n_i are nodes

Geometric operators : area and volume become operators

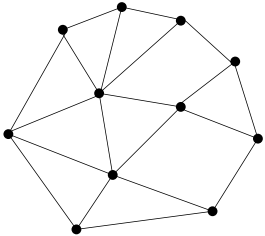
▷ Area acts on edges and Volume on vertices

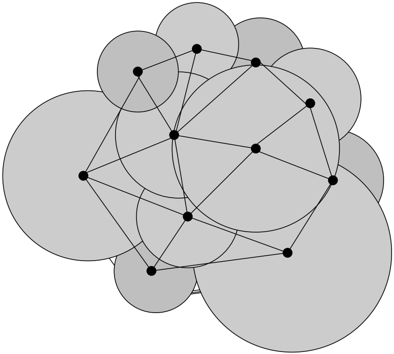


$$\mathcal{A}(\mathcal{S})|S\rangle \; = \; \frac{8\pi\gamma\hbar G}{c^3} \sum_{P\in \mathcal{S}\cap\Gamma} \sqrt{j_P(j_P+1)}|S\rangle$$

▷ The spectra are discrete : existence of a minimal length

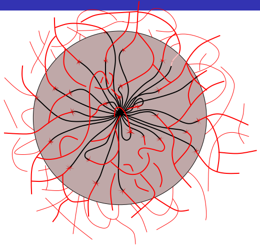
From the kinematics, Space is discrete...





▷ Edges carry quanta of area, nodes carry quanta of volume

Heuristic picture : the Rovelli model



$a_H = 8\pi\gamma\ell_P^2 \sum_j \sqrt{j(j+1)}$

Edges crossing spherical BH

Only spins $1/2$ contribute to the area

- ▷ Number of edges : $a_H = 8\pi\gamma\ell_P^2 \times \mathbf{N} \times \frac{\sqrt{3}}{2}$
- ▷ Number of states : number of singlets in $(1/2)^{\otimes N} \Longrightarrow \mathbf{\Omega} \sim 2^N$
- ▷ Bekenstein-Hawking formula for the entropy when $a_H \gg \ell_P^2$

$$S \;=\; \log(\Omega) \;\sim\; N\log(2) \;=\; \frac{2\log(2)}{8\pi\gamma\ell_P^2\sqrt{3}}a_H \;\Longrightarrow\; \gamma = \frac{\log(2)}{\pi\sqrt{3}}\,.$$

Refined models : all spins contribute

- ▷ The value of γ changes. Is γ relevant at the quantum level?

Relation to $SU(2)$ Chern-Simons theory (Rovelli - Engle,KN,Perez)

Hamiltonian description of Black Holes

- ▷ Governed by a Chern-Simons theory

$$S(A) = \frac{k}{4\pi}\int \langle A \wedge dA + \frac{2}{3} A \wedge A \wedge A \rangle$$

- ▷ The coupling constant (the level) $k \propto a_H$ and $\langle , \rangle$ is a trace
- ▷ Classical solutions : flat connections with singularities at punctures

Quantization of a CS theory on a punctured sphere

- ▷ Hilbert space is the space of (q-deformed) $SU(2)$ intertwiners

$$\mathcal{H}(j_1, \cdots, j_n) \; = \; \text{Inv}(V_{j_1} \otimes \cdots \otimes V_{j_N}) \, .$$

- ▷ Closed formula for the dimension

$$\mathcal{Z} = \frac{2}{k+2} \sum_{d=1}^{k+1} \sin^2(\frac{\pi d}{k+2}) \prod_{i=1}^{N} \frac{\sin(\frac{\pi d}{k+2}(2j_i+1))}{\sin(\frac{\pi d}{k+2})} \, .$$

- ▷ One recovers the BH entropy and log corrections

Going to complex variables (Frodden, Geiller, KN, Perez)

Analytic continuation to $\gamma = i$

- ▷ The level k becomes imaginary and $\lambda = |k|$
- ▷ New partition function for CS theory

$$\mathcal{Z} \simeq \frac{2}{\lambda} \sum_{d=1}^{\lambda} \sinh^2\left(\frac{\pi d}{\lambda}\right) \prod_{i=1}^{N} \frac{\sinh\left(\frac{\pi d}{\lambda}(2j_i+1)\right)}{\sinh\left(\frac{\pi d}{\lambda}\right)} \, .$$

- ▷ It should correspond to CS theory with $SL(2, \mathbb{C})$ gauge group

Semi-classical limit

- ▷ large spin $j_i \to \infty$ and $\ell_P \to 0$ s.t. $\ell_P^2 j_i \to \ell_i$
- ▷ $\log \mathcal{Z} \sim a_H / 4\ell_P^2$ with $a_H = 8\pi \sum_i \ell_i$

Energy of the vacuum and temperature of excited states

Beyond the leading order term

▷ The partition function takes the form

$$\mathcal{Z} \simeq \frac{2\sinh^2\pi}{\lambda} \prod_{i=1}^{N} \left(\sum_{m=0}^{\infty} \exp(-\beta E_m^{(j_i)}) \right)$$

▷ The energy spectrum is the energy of an accelerated observer (same as E. Bianchi)

$$E_m^{(j)} = \langle j, m | aK | j, m \rangle = (m-j)a$$

▷ The temperature in the Unruh temperature $\beta = 2\pi/a$ of the observer

Locally at the vicinity of the quantum Black Hole

▷ Thermalized states at β : probably gravitational dof

▷ The vacuum has a negative energy and responsible for the huge entropy

Conclusion and references

LQG in a nut shell

- ▷ Kinematical States are labelled by topological graphs
- ▷ Geometrical operators have discrete spectra
- ▷ The quantum dynamics is still under construction

What do Quantum Black Holes teach us ?

- ▷ The Barbero-Immirzi parameter should return to $\gamma = i$
- ▷ The LQG dof contain the gravitons (at least close to a BH horizon)

Effective description of a Quantum Black Hole

- ▷ Thermalized graviton at Unruh temperature
- ▷ The vacuum has a negative energy
- ▷ The energy of the vacuum is responsible for the entropy

Our recent references

- ▷ Engle,KN,Perez : PRL105.031302 (2009) - JHEP 1105 (2011)
- ▷ Frodden,Geiller,KN,Perez :arXiv :1212.4060 - JHEP 05 139 (2013)
- ▷ Ghosh,KN,Perez : arXiv :1309.4563