Black Holes in Loop Quantum Gravity

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**Introduction**

**Quantizing Gravity: Why?**

Two (amazingly efficient) fundamental theories
- Small scales (up to ...) : Quantum Physics and QFT
- Large scales (up to ...) : General Relativity

When General Relativity meets Quantum Physics
- Origin of the Universe: below the Planck length \( \ell_P = \sqrt{\frac{\hbar G}{c^3}} \)

\[
rs = \frac{2Gm}{c^2} \sim \frac{\hbar}{mc} = \lambda_c
\]

- Black Holes: Bekenstein-Hawking thermodynamics \( S = a_H/4\ell_P^2 \)
- Problem of Singularities in General Relativity: Penrose

Problems in Quantizing Gravity
- Perturbative quantization: non-renormalizable theory
- Hamiltonian quantization: technically too involved
- What is the meaning of quantizing space-time?
Introduction

Quantizing Gravity : How ?

(Inequivalent) Attempts to quantize gravity

▷ String theory : General Relativity should be modified at high energies but quantization rules remain unchanged

▷ Loop Quantum Gravity : GR is fundamental but the quantization process should be modified/adapted

▷ Others : Dynamical triangulations, Non-Commutative Geometry ...

Loop Quantum Gravity : a very powerful approach

▷ The Space is fundamentally discrete

▷ The fundamental length is a UV cutt-off : consequences in LQC

▷ Discreetness provides a statistical explanation of BH thermodynamics

Even though the theory is not yet complete

▷ The kinematics is totally well understood : states and observables

▷ But the quantum dynamics is not fully under control : Spin-Foams

▷ Many open questions concerning the classical limit
Black Holes to probe quantum gravity at the Planck length

Black Hole thermodynamics: the classical viewpoint

- Quantum Field Theory in a classical curved background
- Bekenstein-Hawking entropy for any Black Holes: $S = a_H/4\ell_P^2$
- Where $a_H$ is the horizon area: information is contained in the area
- Thermal radiation of particles at Hawking temperature $T = \kappa/2\pi$

Quantum Black Holes: old results

- Precise description of the BH micro states: partition function $Z$
- The micro canonical entropy $S = \log(Z)$
- It reproduces the classical result at the semi-classical limit

Complex Quantum Black Holes: new results

- Effective description for an observer close to a Quantum BH
- Particles thermalized at temperature $T$: graviton?
- The spectrum: $|j, m\rangle$ where $m = -j, -j + 1, \cdots$ and $E_m = mE_0$
- The BH entropy is the entropy of the vacuum
- The BH temperature is obtained from the excited states
Overview

1. Loop Quantum Gravity in a nut shell
   - Why standard quantization schemes fail?
   - From Ashtekar gravity...
   - ... To kinematical quantum states
   - Physical interpretation: discrete geometry

2. Black Holes in LQG: a quick review
   - Heuristic picture: the Rovelli model
   - Relation to Chern-Simons theory

3. New results: vacuum, temperature and gravitons
   - Going to complex variables
   - The new partition function
   - Vacuum and entropy; Excited states and temperature
Loop Quantum Gravity in a nut shell

Why standard quantization schemes fail?

Lagrangian formulation: $\mathcal{M}$ is the 4D space-time

- Einstein-Hilbert action: functional of the metric $g$

\[
S_{EH}[g] = \int d^4x \sqrt{|g|} R
\]

Hamiltonian formulation: $\mathcal{M} = \Sigma \times \mathbb{R}$ ('61)

- ADM variables: $ds^2 = N^2 dt^2 - (N^a dt + h_{ab} dx^b)(N^a dt + h_{ac} dx^c)$
- ADM action: $(h, \pi)$ canonical variables

\[
S_{ADM}[h, \pi; N, N^a] = \int dt \int d^3x (\dot{h} \pi + N^a H_a[h, \pi] + NH[h, \pi])
\]

- Constraints $H = 0 = H_a$ generate the diffeomorphisms

What about the quantization?

- Path integral: non renormalizable theory
- Hamiltonian: too complicated constraints!
Starting point: first order formulation of gravity

- A tetrad $e^I_{\mu}$ (4 x 4 matrix) such that $g_{\mu\nu} = e^I_{\mu} e^J_{\nu} \eta_{IJ}$
- A so(3,1) spin-connection $\omega^I_{\mu}$ related to Levi-Civitta connection

The Ashtekar variables (’86)

- New complex variables: $E^a = \epsilon^{abc} e_b \times e_c$ and $A^i_a = \omega^i_a + \gamma \omega^0_a$
- Pair of canonical variables:

$$\{A^i_a(x), E^b_j(y)\} = (8\pi \gamma G) \delta^b_a \delta^i_j \delta^3(x, y)$$

- Where $\gamma = \pm i$: Complex (or non-compact) symmetry group
- The constraints become polynomials in $E$ and $A$
- But... No one knows how to deal with complex variables

Immirzi-Barbero parameter $\gamma$

- One considers $\gamma$ real: canonical transformation
- Interpret as a Wick rotation: gauge group becomes compact $SU(2)$
Schrodinger like quantization

States are functionals $\Psi(A)$ of the connection $A$

$$\hat{E} \triangleright \Psi(A) = i\gamma \ell_P \frac{\delta \Psi}{\delta A} \quad \text{and} \quad \hat{A} \triangleright \Psi(A) = A\Psi(A).$$

But no measure and no scalar product exists, then no predictions

Polymer quantization

States have support on the one dimensional lines of a graph $\Gamma$

Fundamental variables form the holonomy-flux algebra associated to edges $e$ of $\Gamma$ and surfaces $S$ dual to $\Gamma$

$$A(e) = P \exp(\int_e A) \quad \text{and} \quad E_f(S) = \int_S \text{Tr}(f \star E).$$

Cylindrical functions : $f \in \text{Cyl}(\Gamma)$ is a function of $A(e) \in SU(2)$

$E_f(S)$ acts as a vector field on $f$ if $S \cap \gamma \neq 0$. 
Loop Quantum Gravity in a nut shell

Physical interpretation (Rovelli - Smolin)

**Kinematical states : basis of spin-networks**

- They are generalizations of Wilson loops with nodes

![Diagram of spin-networks]

- $\ell_i$ are oriented links
- $n_i$ are nodes

**Geometric operators : area and volume become operators**

- Area acts on edges and Volume on vertices

\[
\mathcal{A}(S)|S\rangle = \frac{8\pi \gamma \hbar \kappa}{c^3} \sum_{P \in S \cap \Gamma} \sqrt{j_P(j_P + 1)}|S\rangle
\]

- The spectra are discrete : existence of a minimal length
From the kinematics, Space is discrete...

Edges carry quanta of area, nodes carry quanta of volume
Black Holes in LQG: a quick review

**Heuristic picture: the Rovelli model**

\[ a_H = 8\pi\gamma\ell_P^2 \sum_j \sqrt{j(j+1)} \]

Edges crossing spherical BH

**Only spins 1/2 contribute to the area**

- Number of edges: \( a_H = 8\pi\gamma\ell_P^2 \times N \times \frac{\sqrt{3}}{2} \)
- Number of states: number of singlets in \((1/2)^\otimes N \implies \Omega \sim 2^N\)
- Bekenstein-Hawking formula for the entropy when \( a_H \gg \ell_P^2 \)

\[
S = \log(\Omega) \sim N \log(2) = \frac{2 \log(2)}{8\pi\gamma\ell_P^2 \sqrt{3}} a_H \implies \gamma = \frac{\log(2)}{\pi \sqrt{3}}.
\]

**Refined models: all spins contribute**

- The value of \( \gamma \) changes. Is \( \gamma \) relevant at the quantum level?
Hamiltonian description of Black Holes

▷ Governed by a Chern-Simons theory

\[ S(A) = \frac{k}{4\pi} \int \langle A \wedge dA + \frac{2}{3} A \wedge A \wedge A \rangle \]

▷ The coupling constant (the level) \( k \propto a_H \) and \( \langle , \rangle \) is a trace

▷ Classical solutions: flat connections with singularities at punctures

Quantization of a CS theory on a punctured sphere

▷ Hilbert space is the space of \((q\)-deformed\) \(SU(2)\) intertwiners

\[ \mathcal{H}(j_1, \cdots, j_n) = \text{Inv}(V_{j_1} \otimes \cdots \otimes V_{j_N}) \]

▷ Closed formula for the dimension

\[ Z = \frac{2}{k+2} \sum_{d=1}^{k+1} \sin^2 \left( \frac{\pi d}{k+2} \right) \prod_{i=1}^{N} \frac{\sin \left( \frac{\pi d}{k+2} (2j_i + 1) \right)}{\sin \left( \frac{\pi d}{k+2} \right)} . \]

▷ One recovers the BH entropy and log corrections
Analytic continuation to $\gamma = i$

- The level $k$ becomes imaginary and $\lambda = |k|$
- New partition function for CS theory

$$Z \simeq \frac{2}{\lambda} \sum_{d=1}^{\lambda} \sinh^2\left(\frac{\pi d}{\lambda}\right) \prod_{i=1}^{N} \frac{\sinh\left(\frac{\pi d}{\lambda}(2j_i + 1)\right)}{\sinh\left(\frac{\pi d}{\lambda}\right)}.$$ 

- It should correspond to CS theory with $SL(2, \mathbb{C})$ gauge group

Semi-classical limit

- large spin $j_i \to \infty$ and $\ell_P \to 0$ s.t. $\ell_P^2 j_i \to \ell_i$
- $\log Z \sim a_H/4\ell_P^2$ with $a_H = 8\pi \sum_i \ell_i$
Beyond the leading order term

- The partition function takes the form

\[ Z \simeq \frac{2 \sinh^2 \pi}{\lambda} \prod_{i=1}^{N} \left( \sum_{m=0}^{\infty} \exp(-\beta E_{m}^{(j_i)}) \right) \]

- The energy spectrum is the energy of an accelerated observer (same as E. Bianchi)

\[ E_{m}^{(j)} = \langle j, m | aK | j, m \rangle = (m - j)a \]

- The temperature in the Unruh temperature \( \beta = \frac{2\pi}{a} \) of the observer

Locally at the vicinity of the quantum Black Hole

- Thermalized states at \( \beta \): probably gravitational dof

- The vacuum has a negative energy and responsible for the huge entropy
Conclusion and references

**LQG in a nut shell**
- Kinematic States are labelled by topological graphs
- Geometrical operators have discrete spectra
- The quantum dynamics is still under construction

**What do Quantum Black Holes teach us?**
- The Barbero-Immirzi parameter should return to $\gamma = i$
- The LQG dof contain the gravitons (at least close to a BH horizon)

**Effective description of a Quantum Black Hole**
- Thermalized graviton at Unruh temperature
- The vacuum has a negative energy
- The energy of the vacuum is responsible for the entropy

**Our recent references**