Quantum spindown of highly magnetized neutron stars

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Outline

1. Electromagnetic properties of vacuum
   - Euler-Heisenberg Lagrangian
   - Vacuum magnetization
2. Quantum Vacuum Friction (QVF)
   - QVF in neutron stars
   - Influence on pulsars spindown evolution
3. Observational consequences
   - Constraints on the equation of state
   - Population synthesis

Electromagnetic properties of vacuum

$\rightarrow$ See talk of R. Battesti

Quantum corrections to QED

Maxwell theory

- Lorentz invariants ($c = 1$)

$$\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{\mathbf{B}^2 - \mathbf{E}^2}{2} \quad , \quad \mathcal{G} = \frac{1}{4} F_{\mu\nu} \, {}^{\star}F^{\mu\nu} = -\mathbf{E} \cdot \mathbf{B}$$

- Lagrangian $\quad \rightarrow \quad \mathcal{L}_0 = -\frac{\mathcal{F}}{\mu_0}$

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- Lagrangian $\rightarrow$ $\mathcal{L}_0 = -\frac{\mathcal{F}}{\mu_0}$

Quantum corrections in high EM field

- Critical field $B_c = \frac{m^2 c^2}{e\hbar} \sim 4 \times 10^9 \, \mathrm{T} = 4 \times 10^{13} \, \mathrm{G}$.
  - $\rightarrow$ Enough energy to create virtual electron-positron pairs.
  - $\rightarrow$ EM field aquires a (virtual) charge distribution.
- Non linear (quantum) correction to the Lagragian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$

- Effective Lagrangian at one-loop

$$\mathcal{L}_1 = 2\alpha \frac{B_c^2}{\mu_0} \int_0^\infty \frac{\mathrm{d}s}{s^3} e^{-is} \left\{ s^2 \frac{|\mathcal{G}|}{B_c^2} \cot\left(s \frac{(\sqrt{\mathcal{F}^2+\mathcal{G}^2}+\mathcal{F})^{1/2}}{B_c} \right) \coth\left(s \frac{(\sqrt{\mathcal{F}^2+\mathcal{G}^2}-\mathcal{F})^{1/2}}{B_c} \right) + s^2 \frac{2\mathcal{F}}{3B_c^2} - 1 \right\}$$

$\rightarrow$ α the fine structure constant.

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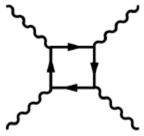
Example of weak field approximation

$$\mathcal{L}_1 \simeq \alpha \frac{B_c^2}{\mu_0} \left(\frac{8}{45} \frac{\mathcal{F}^2}{B_c^4} + \frac{14}{45} \frac{\mathcal{G}^2}{B_c^4} \right) \propto (\mathbf{E}^2 - \mathbf{B}^2)^2 + 7(\mathbf{E} \cdot \mathbf{B})^2$$

→ 4-wave mixing phenomenology

→ Not yet tested experimentally
→ Battesti & Rizzo Rep. Prog. Phys. 2013



- Varying the action $S = \int (\mathcal{L}_0 + \mathcal{L}_1) d^4x$ with respect to A_μ and $\partial_\nu A_\mu$ gives modified Maxwell equations

$$\begin{aligned} \nabla \cdot \left(\epsilon_0 \mathbf{E} + 2 \frac{\partial \mathcal{L}_1}{\partial \mathbf{E}} \right) &= 0 \\ \nabla \times \left(\frac{\mathbf{B}}{\mu_0} - 2 \frac{\partial \mathcal{L}_1}{\partial \mathbf{B}} \right) &= \frac{\partial}{\partial t} \left(\epsilon_0 \mathbf{E} + 2 \frac{\partial \mathcal{L}_1}{\mathbf{E}} \right) \end{aligned}$$

- Effective (non-linear) medium with

$$\mathbf{P}_{qv} \equiv 2 \frac{\partial \mathcal{L}_1}{\partial \mathbf{E}} \quad \text{and} \quad \mathbf{M}_{qv} \equiv 2 \frac{\partial \mathcal{L}_1}{\partial \mathbf{B}}$$

- $\mathcal{G} \to 0$ and $\xi = \dfrac{B}{B_c}$

$$\mathcal{L}_1 = \frac{2\alpha}{\mu_0} B_c^2 \left\{ \frac{1}{4} + \left(\frac{1}{2} - \xi + \frac{1}{3}\xi^2 \right) \ln 2\xi - \frac{1}{3}\xi^2 + 4\xi^2 \zeta' \left(-1, \frac{1}{2\xi} \right) \right\}$$

  $\rightarrow$ Still valid in the presence of an electric field if $\mathbf{E} \cdot \mathbf{B} = 0$ (example of a force free magnetosphere)

- Magnetic susceptibility of vacuum

$$\mathbf{M}_{qv} = \chi(\xi) \frac{\mathbf{B}}{\mu_0} \quad , \quad \chi(\xi) = \frac{2\mu_0}{B_c^2} \frac{\mathcal{L}_1'(\xi)}{\xi}$$

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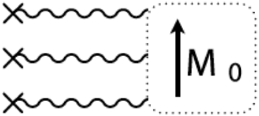
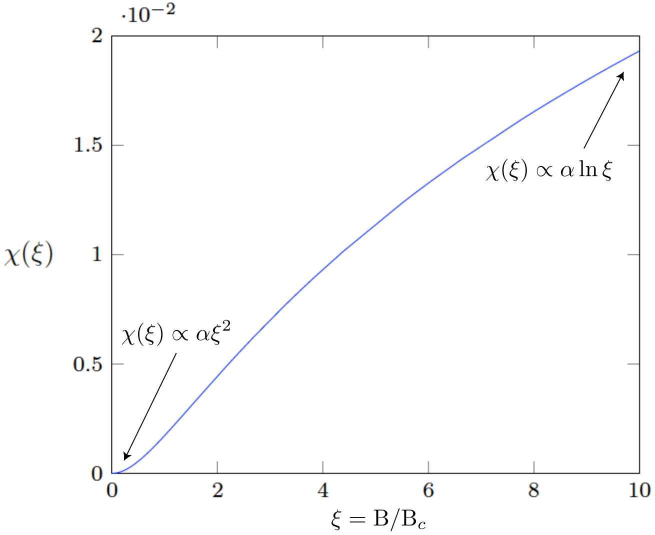
- Weak-field limit

$$\chi(\xi) \underset{\xi \ll 1}{\sim} \frac{16}{45} \alpha \xi^2$$



Magnetization in vacuum

Static effects in a pure magnetic field



Quantum Vacuum Friction

Dupays, EPL 2008, EPL 2012

- In our model a neutron star is a compact object with mass $M \sim 1.4 M_{\odot}$, a radius $R \sim 10\,\mathrm{km}$ and a large magnetic field ($B_0 \sim 10^{10} - 10^{14}\,\mathrm{G}$).

NS = Fast-rotating magnetic dipole moment

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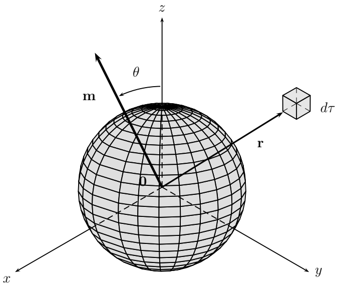
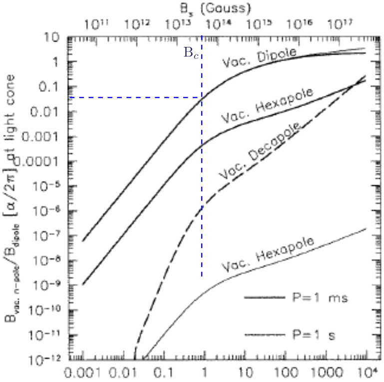
- The magnetic dipole moment is tilted with respect to the rotational axis.

Non static effects from the vacuum + backreaction

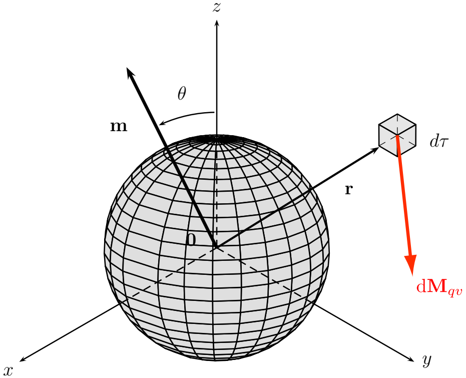




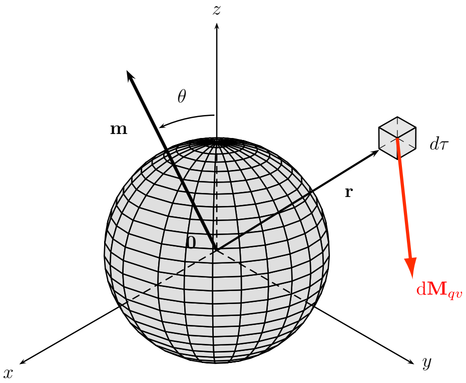
(from Heyl J. Phys. A 30 (1997) 6475)

$$\mathbf{B}(\mathbf{r},t) \simeq \left(\frac{\mu_0}{4\pi}\right)\left[\frac{3(\mathbf{m}(t-r/c)\cdot\mathbf{r})\mathbf{r}}{r^5} - \frac{\mathbf{m}(t-r/c)}{r^3}\right] \qquad (kr \ll 1)$$

Toy model
Energy loss rate



- $\mathbf{m}(t - r/c) \rightarrow \mathbf{B}(\mathbf{r}, t) \xrightarrow{EH} \mathrm{d}\mathbf{M}_{qv}(\mathbf{r}, t) \rightarrow \mathrm{d}\mathbf{B}_{qv}(\mathbf{0}, t + r/c)$



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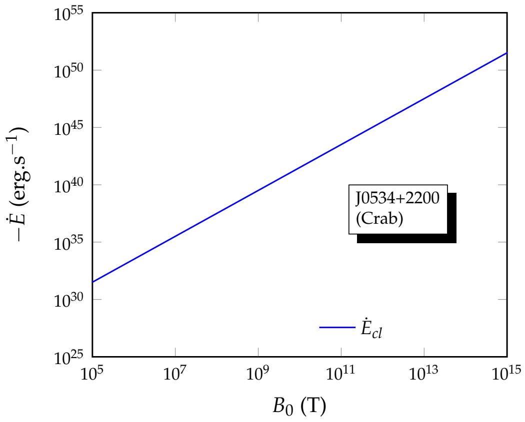
- Braking torque $\Rightarrow$ loss of energy

$$d\dot{E}_{qv} = -\left(\mathbf{m}(t + r/c) \times \mathbf{dB}_{qv}(\mathbf{0}, t + r/c)\right)\Omega \cdot \mathbf{u_z} \qquad \left(\Omega = \frac{2\pi}{P}\right)$$

Energy loss rate
Classical versus quantum

$$\dot{E} = \underbrace{\left(\frac{128\pi^5}{3}\right) \frac{\sin^2\theta}{\mu_0 c^3} \frac{B_0^2 R^6}{P^4}}_{\dot{E}_{cl}\,(\text{classical model})}$$



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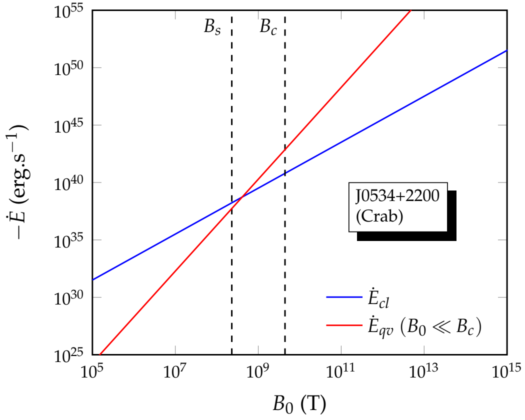

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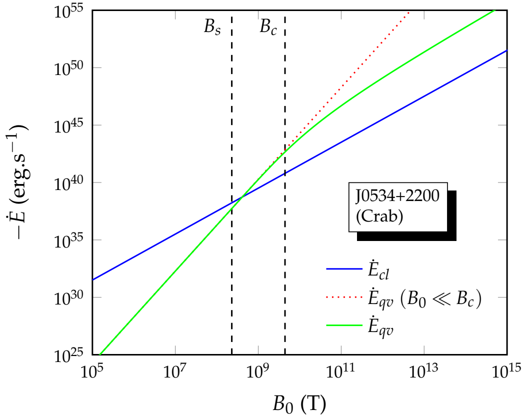

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- Different scaling with respect to the period $P = 2\pi/\Omega$.

$\rightarrow$ $\dot{E}_{cl} \sim \dot{E}_{qv}$ when $\dfrac{B_0}{B_c} \sim \dfrac{R\Omega}{c}\dfrac{1}{\sqrt{\alpha}}$

$\rightarrow$ Late-time domination of the quantum effect (even if $B_0 \ll B_c$).

$$\dot{E} = \underbrace{\frac{3}{2}\left(\frac{128\pi^5}{3}\right)\frac{1+\sin^2\theta}{\mu_0 c^3}\frac{B_0^2 R^6}{P^4}}_{\dot{E}_{cl}(\text{magnetospheric model})} + \underbrace{\alpha\left(\frac{512\pi^3}{75}\right)\frac{\sin^2\theta}{\mu_0 c}\frac{B_0^2 R^4}{P^2}\frac{B_0^2}{B_c^2}}_{\dot{E}_{qv}(\text{QVF for } B_0 \ll B_c)}$$

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- Effect of the magnetosphere (Spitkovsky ApJ 2006).
- What about another classical effect due to the magnetosphere scaling as P^{-2} ?

  $\rightarrow$ Mainly suppressed by co-rotation ! [work in progress]

QVF influence on pulsars spindown evolution

$P - \dot{P}$ relation

$$E = E_{\text{kin}} = \frac{1}{2} J \Omega^2 \quad \Rightarrow \quad \dot{P} = \frac{\mathcal{T}_{cl}}{P} + \frac{P}{\mathcal{T}_{qv}}$$

$$\begin{aligned}
\mathcal{T}_{cl} &\simeq 1.3 \times 10^{-15}\,\mathrm{s} \left(\frac{B_0}{10^8\,\mathrm{T}} \right)^2 (1 + \sin^2\theta) \left(\frac{R}{10\,\mathrm{km}} \right)^4 \left(\frac{1.4\,\mathrm{M}_\odot}{M} \right) \\
\mathcal{T}_{qv} &\simeq 2.1 \times 10^{13}\,\mathrm{s} \left(\frac{10^8\,\mathrm{T}}{B_0} \right)^4 \frac{1}{\sin^2\theta} \left(\frac{10\,\mathrm{km}}{R} \right)^2 \left(\frac{M}{1.4\,\mathrm{M}_\odot} \right)
\end{aligned}$$

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- Quantum domination when

$$P > \sqrt{\mathcal{T}_{cl}\mathcal{T}_{qu}} \simeq 140\,\text{ms} \left(\frac{10^8\,\text{T}}{B_0}\right) \left(\frac{R}{10\,\text{km}}\right) \sqrt{\frac{1 + \sin^2\theta}{\sin^2\theta}}$$

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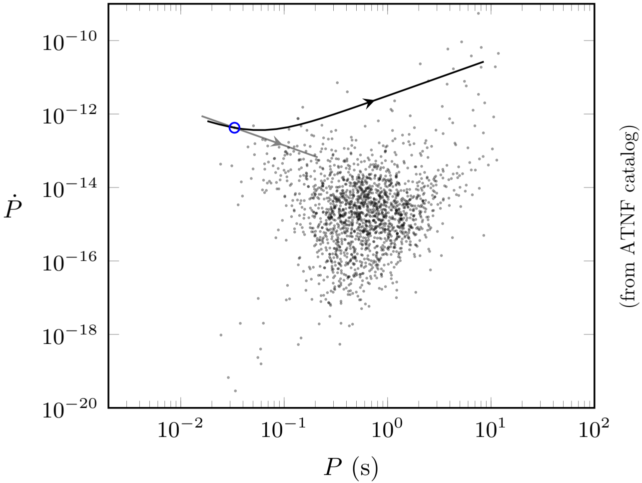
- $\mathcal{T}_{cl}$ and $\mathcal{T}_{qv}$ can be determined if one measures P, $\dot{P}$ and $\ddot{P}$.
  $\rightarrow$ will give constraints on $M, R, B_0, \sin\theta$

- Simulation over 50 kyr, $\dot{P} = \frac{\mathcal{T}_{cl}}{P} + \frac{P}{\mathcal{T}_{qv}}$



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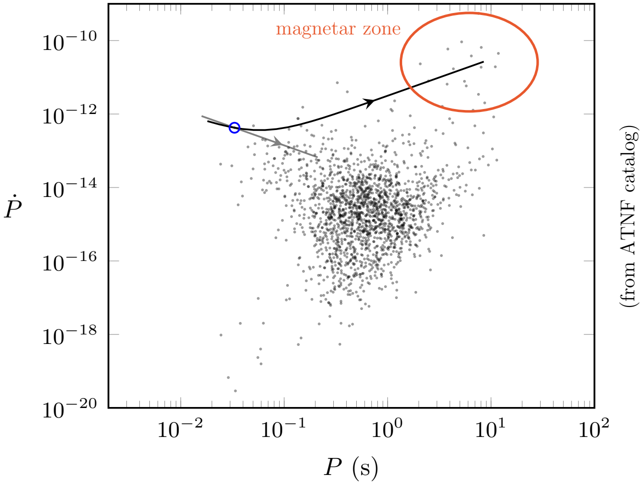
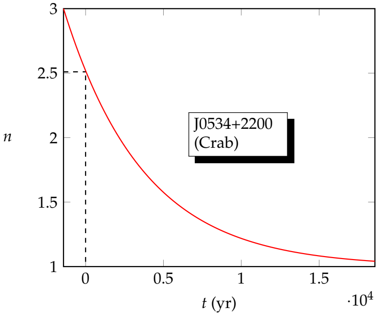


- Braking index : $\dot{\Omega} = -k\Omega^n \quad \rightarrow \quad n_0 = \Omega\ddot{\Omega}/\dot{\Omega}^2$
- Classical dipolar model (constant B_0) : $n = 3$
- QVF implies that n_0 evolves dynamically from 3 to 1

Name	n_0	$\nu\,(s^{-1})$
J1846-0258	2.65(1)	3.07
J0534+2200 (crab)	2.51(1)	30.2
J1513-5908	2.839(3)	6.63
J1119-6127	2.91(5)	2.45
J0540-6919	2.140(9)	19.8
J0835-4510 (vela)	1.4(2)	11.2



Evolution of the breaking index

- Prediction for time evolution of braking indice $\dot{n}_0$

$$\dot{n}_0 = -(3 - n_0)(n_0 - 1)\frac{P_1}{P_0}$$

Name	$\dot{n}_0^{\mathrm{obs}}$ $(10^{-12}\,\mathrm{s}^{-1})$	$\dot{n}_0$ $(10^{-12}\,\mathrm{s}^{-1})$
J0534+2200	0.26	-9.3
J1513-5908	-1.2	-3.1

- Probably other physics involved in the Crab (magnetosphere currents, accretion, pulsar wind etc...) ?

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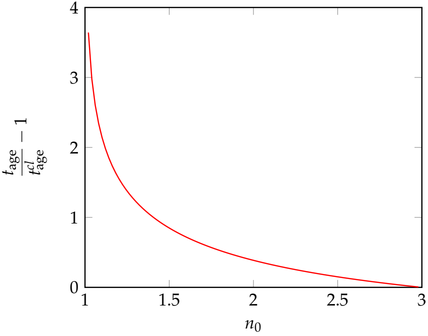
Improving number of confident measure of $\dot{n}_0$ would be necessary

Consequence for the spindown age

$$t_{\text{age}} \underset{P_i \ll P_0}{\sim} \frac{P_0}{(3-n_0)P_1} \ln \frac{n_0 - 1}{2} \quad , \quad t^{cl}_{\text{age}} \underset{P_i \ll P_0}{\sim} \frac{P_0}{2P_1}$$

Name	t_{age} (kyr)	t^{cl}_{age} (kyr)
J1846-0258	.799	.727
J0534+2200 (crab)	1.42	1.24
J1513-5908	1.63	1.56
J1119-6127	1.65	1.61
J0540-6919	2.18	1.67
J0835-4510 (vela)	22.8	11.3



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$$t_{\text{age}} \underset{P_i \ll P_0}{\sim} \frac{P_0}{(3-n_0)P_1} \ln \frac{n_0 - 1}{2} \quad , \quad t_{\text{age}}^{cl} \underset{P_i \ll P_0}{\sim} \frac{P_0}{2P_1}$$

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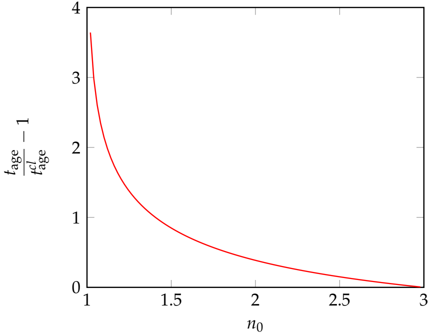


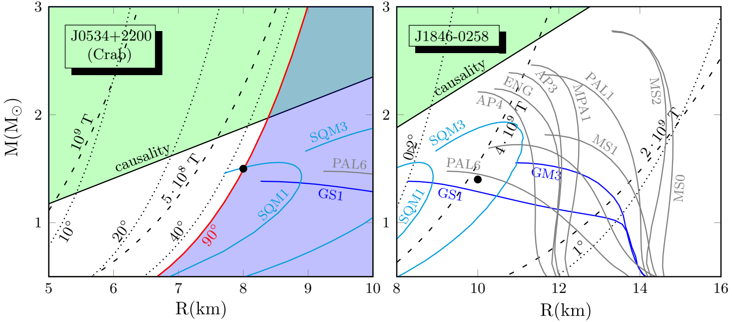
Classical spindown age underestimates the true age
$\rightarrow$ Some observational discrepencies with kinematic and SNR age.

Observational consequences

Work in progress

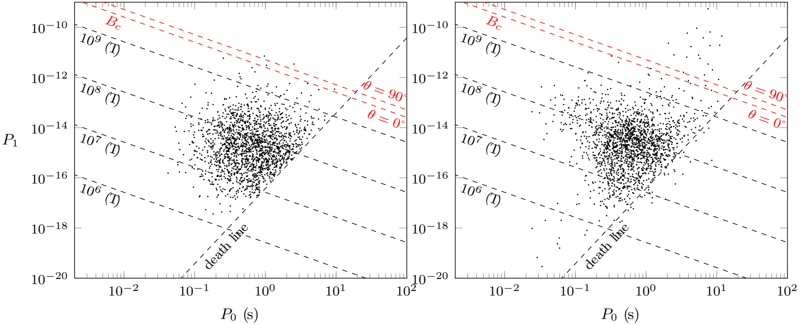
$$B_0 = \left(\frac{\alpha\mu_0 c^3 I(n_0-1)P_0^2 P_1 + \sqrt{\alpha\mu_0 c I P_0 P_1 (\alpha\mu_0 c^5 I (n_0-1)^2 P_0^3 P_1 - 1200\pi^5(3-n_0)B_c^2 R^8)}}{64\pi^3 \alpha R^6 P_0} \right)^{1/2}$$

$$\sin\theta = \left(-\frac{c^2(n_0-1)P_0}{600\pi^5 B_c^2 R^8 (3-n0)} \sqrt{\alpha\mu_0 c I P_0 P_1 (\alpha\mu_0 c^5 I (n_0-1)^2 P_0^3 P_1 - 1200\pi^5 (3-n_0) B_c^2 R^8)} + \frac{\alpha\mu_0 c^5}{600\pi^5 B_c^2 R^8} \frac{(n_0-1)^2}{(3-n0)} P_0^3 P_1 - 1 \right)^{1/2}$$



Population synthesis

- Model of Faucher-Giguere ApJ 2006
- Death line
  - → Radio-quiet when $\frac{B}{P^2} < 0.17 \times 10^8$ T.s^{-2}

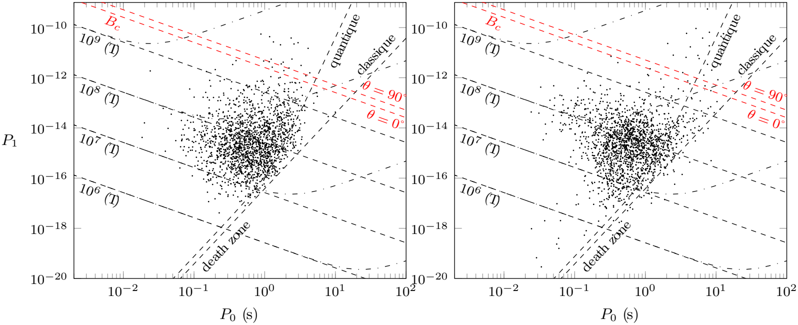


→ The magnetar zone is not explained.

Population synthesis

- Model of Faucher-Giguere ApJ 2006 + quantum evolution
- Death valley

  → Radio-quiet when $\dfrac{B}{P^2} < 0.17 \times 10^8\ \mathrm{T.s}^{-2}$



→ The magnetar zone may be explained [work in progress].

Summary

- Quantum Vacuum Friction is an additional energy loss mechanism that significantly changes the spindown evolution of high magnetized pulsar ($B_0 \gtrsim 10^{11}$ G).
- Confirmation of this effect would be a test of the Euler-Heisenberg Lagrangian.
- Does magnetar really exist ?