The Structure of Neutron-Star Magnetic Fields

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Dan Mazur (1209.4409)

5 November 2013

a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA

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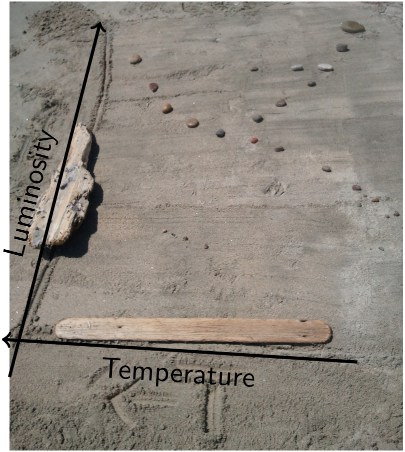

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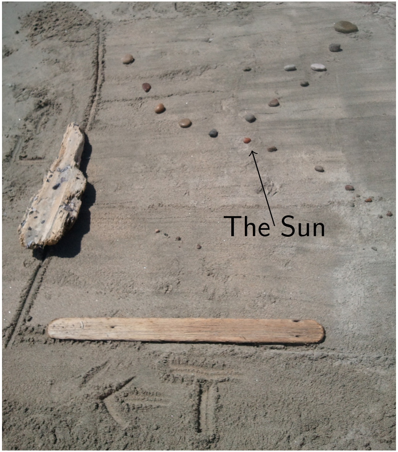

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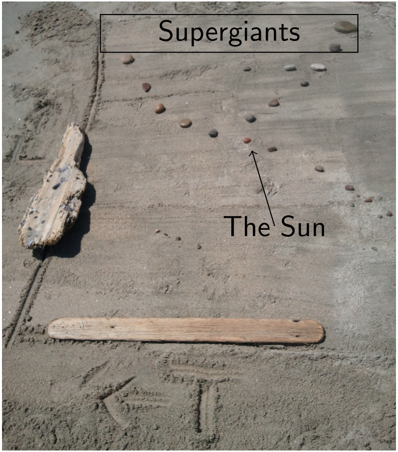

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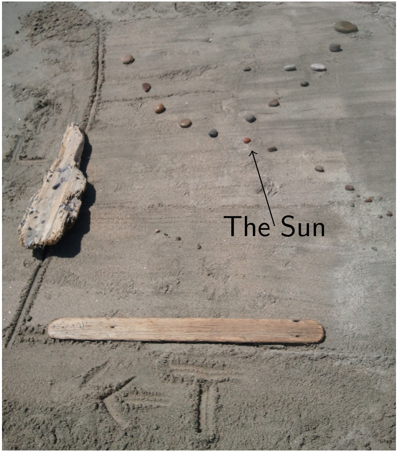
- Supergiants explode and become neutron stars.



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$$R \approx 17\mathrm{km}, M \approx 1.4\mathrm{M}_\odot$$

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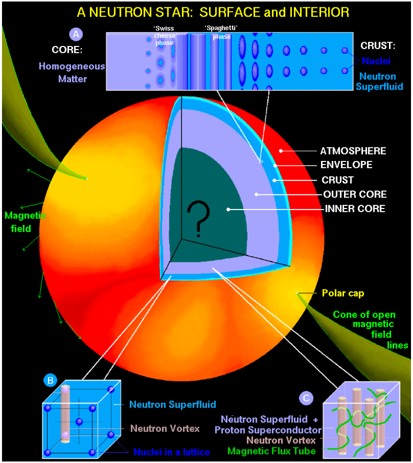

Dany Page

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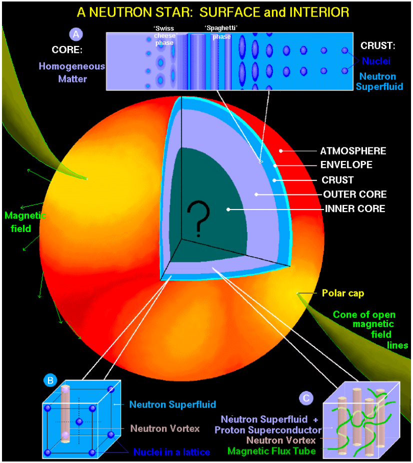

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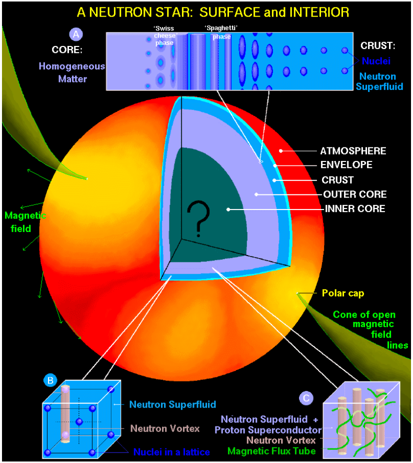

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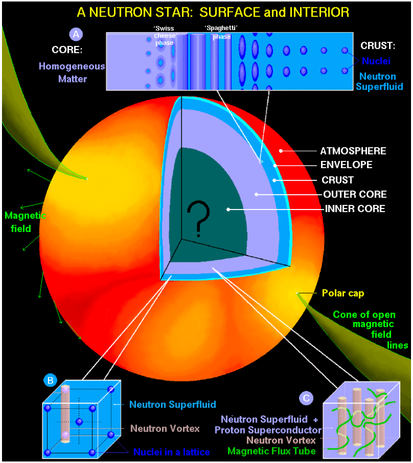

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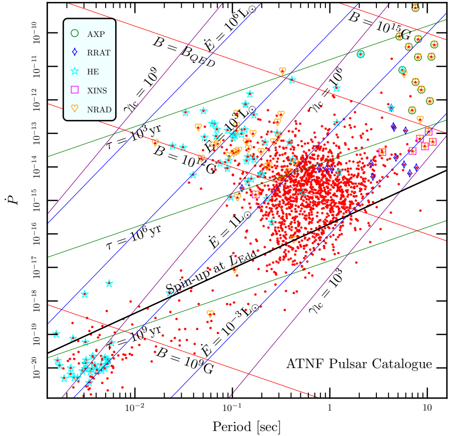
- $B \sim 50\mathrm{G} \ (70000)^2 \sim 10^{11-12}\mathrm{G}$.

Neutron Stars

- The first neutron stars to be identified were radio pulsars.



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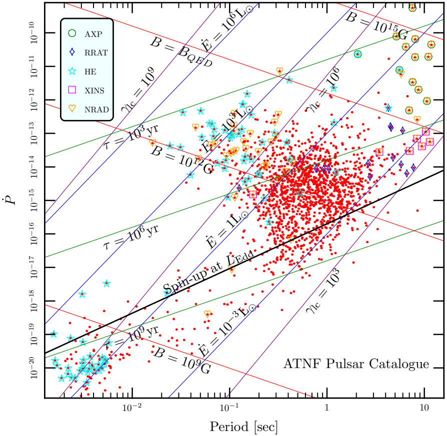

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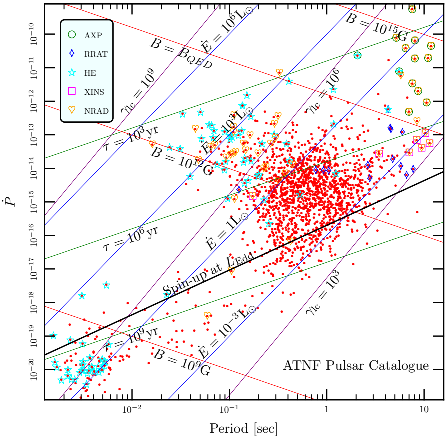
- Lots of flavours – not even including the accretors.



Superconductivity in Neutron Stars

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- Pulsar glitches are well explained by superfluid vortex unpinning in the inner crust of neutron stars (Anderson & Itoh 1975).
- The accelerated cooling of the Cas-A neutron star may indicate a superfluid transition is underway (Elshamouty et al. 2013).

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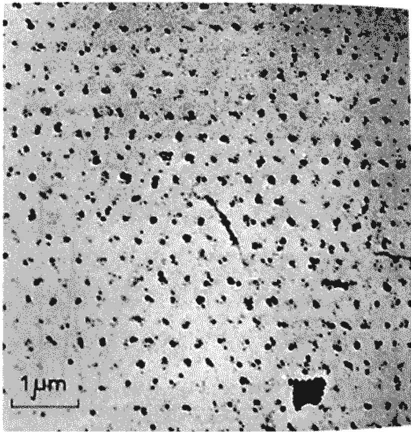

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In a lab superconductor the distance between vortices is $\sim 1\mu$m, and their size is $\sim$ 100nm.

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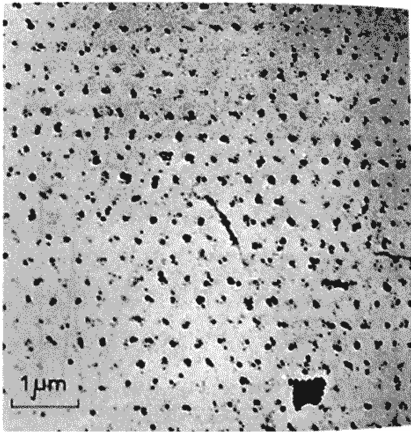

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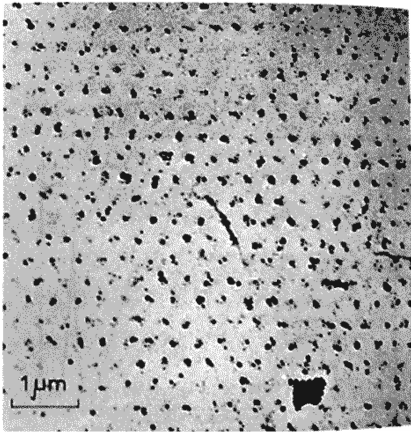

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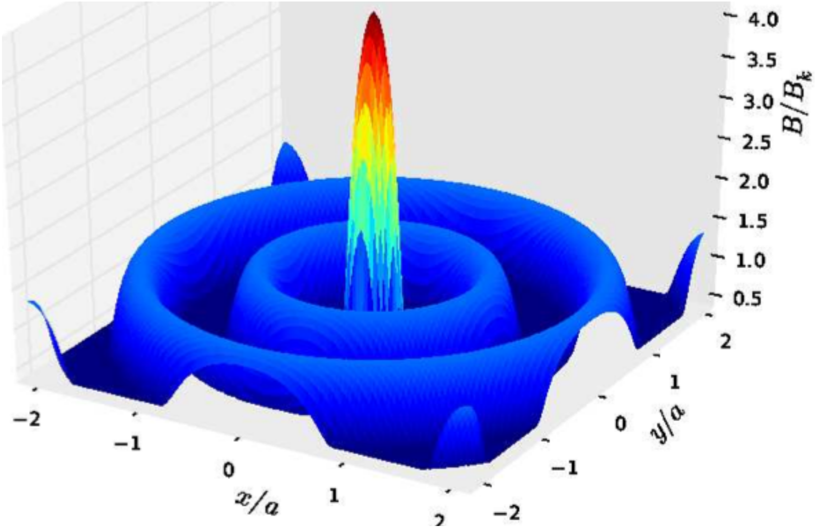
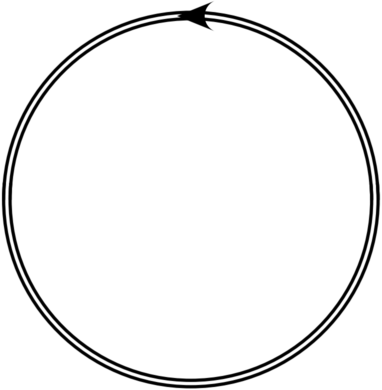
and

$$\lambda_L = \sqrt{\frac{mc^2}{8\pi q^2 n_0}} = 7 \rho_{15} \mathrm{fm}$$

Cylindrical Geometry



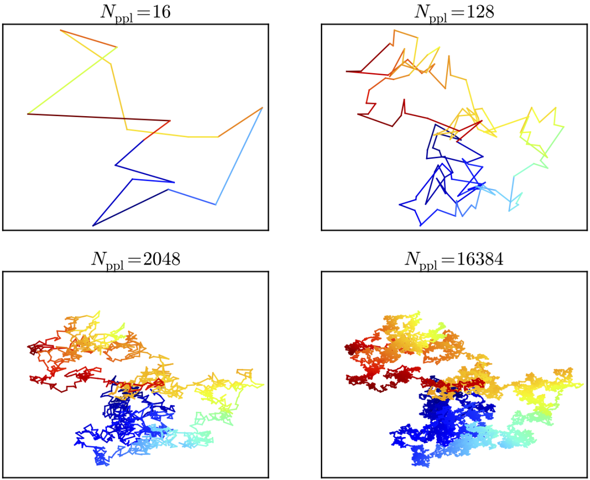
Effective Action



For a magnetic field the effective action is the free energy of the system (actually minus the free energy).

$$\Gamma[A^0_\mu] = \int \mathrm{d}x^4 \left(-\frac{1}{4} F^0_{\mu\nu} F^{0,\mu\nu} \right) - i\hbar \mathrm{Tr} \ln \left[\frac{\not{\Pi} - m}{\not{p} - m} \right]$$

Worldline Numerics — What?



Gies, Roessler, Klingmuller
hep-th/ 0511092
1107.0286

Worldline Numerics — How?

Express the propagator in the proper-time formalism.

$$\Gamma^{(1)}[A_\mu] = \frac{2}{(4\pi)^2}\int_0^\infty \frac{dT}{T^3} e^{-m^2 T}\int d^4 x_{\rm CM} \times \left[\left\langle e^{i\int_0^T d\tau A_\rho(x_{\rm CM}+x(\tau))\dot{x}^\rho(\tau)} \times \frac{1}{4}{\rm tr} e^{\frac{1}{2}\int_0^T d\tau \sigma_{\mu\nu}F^{\mu\nu}(x_{\rm CM}+x(\tau))} \right\rangle_x - 1 \right].$$

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7. Scale the loops for the proper time.

$$\vec{x}(\tau) = \sqrt{T}\vec{y}(\tau/T), \int_0^T d\tau \dot{\vec{x}}^2(\tau) \rightarrow \int_0^1 dt \dot{\vec{y}}^2(t).$$

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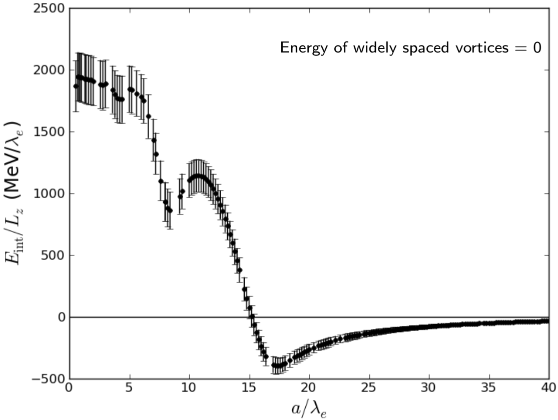
Casimir Force

Discovery of an attractive Casimir interaction



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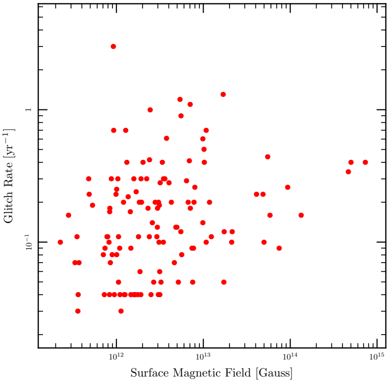
- For stronger fields, the tubes will be evenly and closely packed (like the conventional model).
- These bounds are qualitative as we need to model the superconductor more accurately.

Do we see different glitching as a function of B-field?



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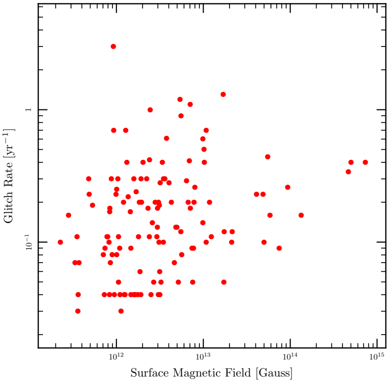
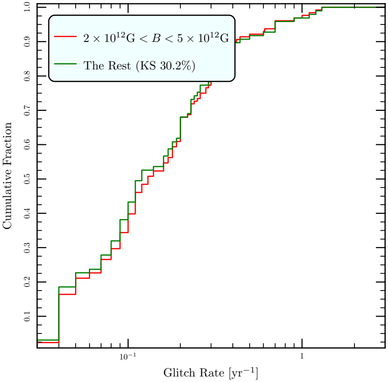


Yu et al. 2013



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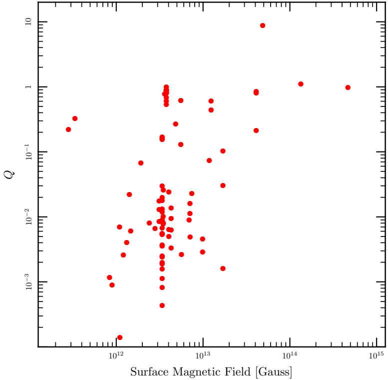

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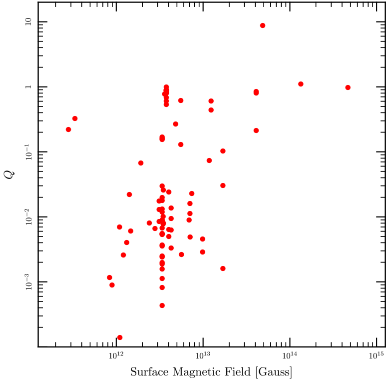

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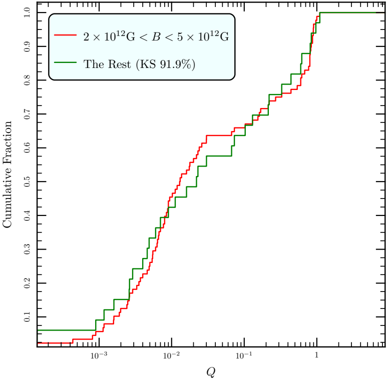

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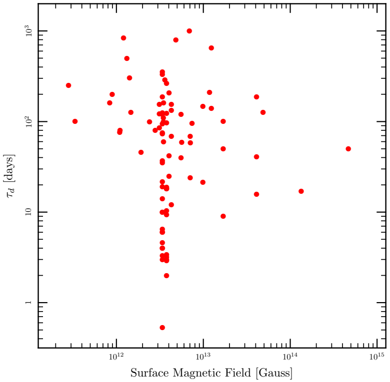

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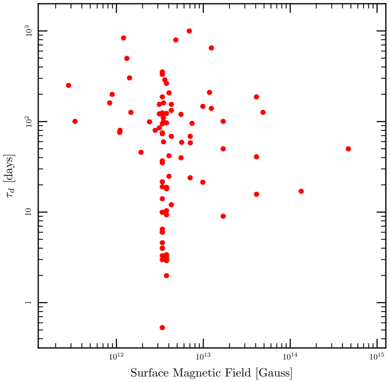

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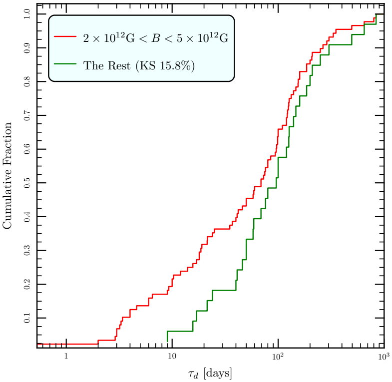

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