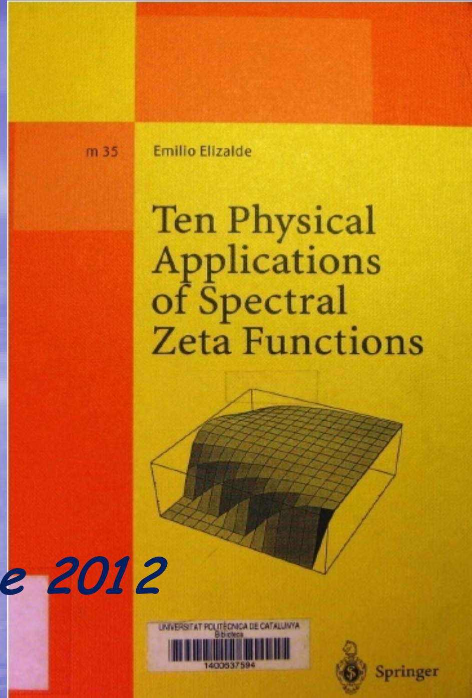
Quantum Vacuum and Cosmology (with a Historical Perspective)

EMILIO ELIZALDE

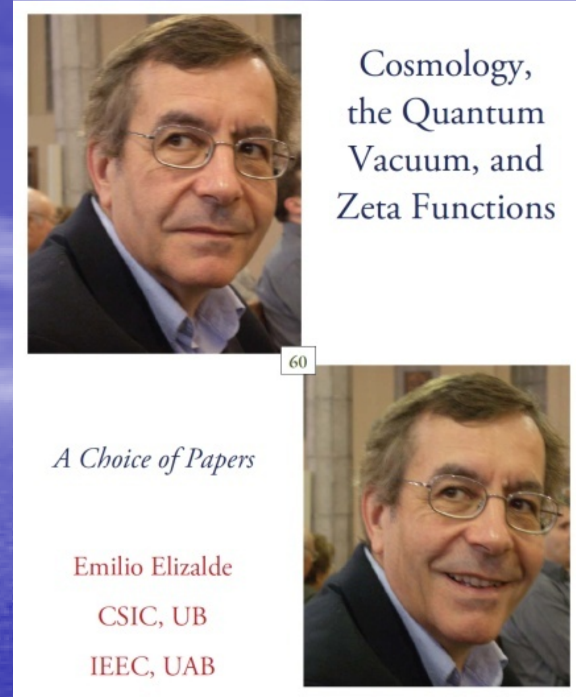
ICE/CSIC & IEEC, UAB, Barcelona

QVG2013: Workshop Quantum Vacuum and Gravitation
4-6 Nov 2013 Toulouse (France)

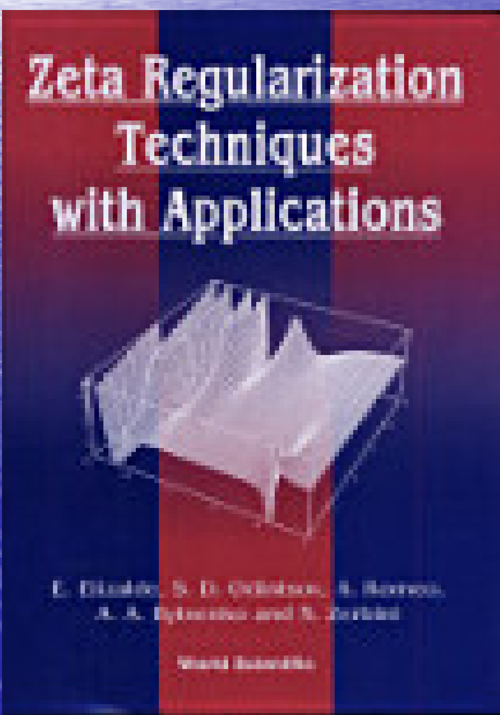
Some Books

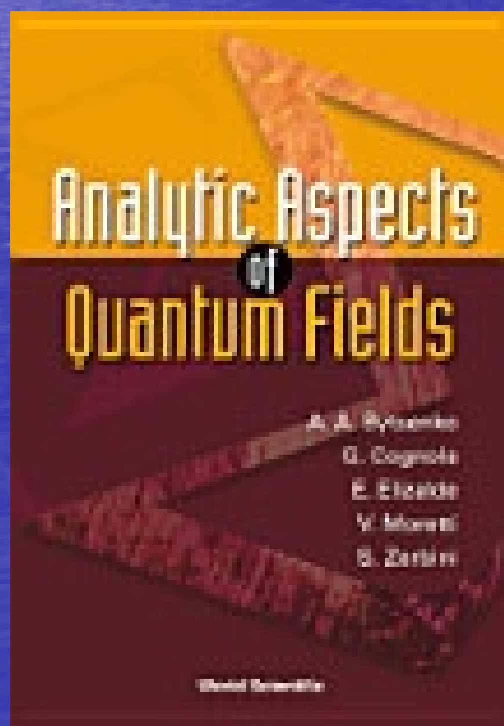


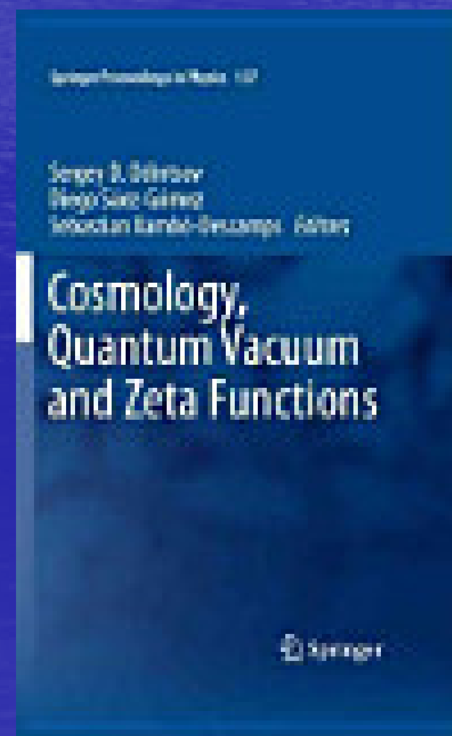
2nd Ed June 2012

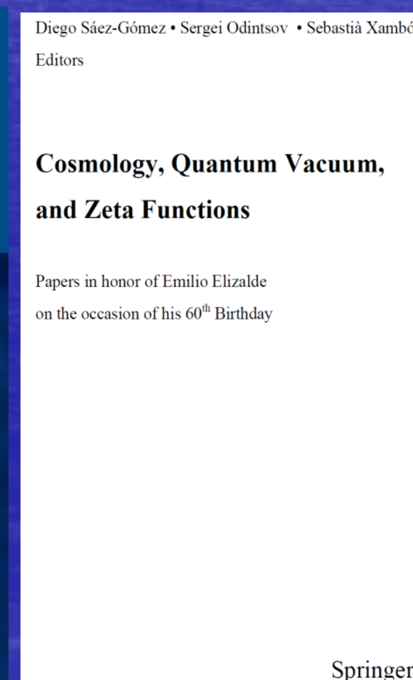


Emilio Elizalde in Moscow, May 20, 2009, during the Fourth International Sakharov Conference on Physics.









Lecture Notes in Physics 855

Emilio Elizalde

Ten Physical Applications of Spectral Zeta Functions

Second Edition

Springer

Instituto de Ciencias del Espacio (ICE/CSIC)

Institut d’Estudis Espacials de Catalunya (IEEC)

SIR

SCIMAGO INSTITUTIONS RANKINGS

SIR World Report 2011 :: Normalized Impact Report

NORMALIZED IMPACT REPORT

This document is an attachment extracted from SIR World Report 2011, it contains exactly the same information but the ordering variable has been set to Normalized Impact. All the values as well as insitutitutions coincide with those included in SIR World Report 2011 :: Global Ranking also available at http://www.scimagoir.com

This report uses two decimal values for the Normalized Impact variable instead just one as it is usual in SIR Reports in order to avoid an extremely high number of identical ranks in institutions. For those institutions which have identical NI values using two decimals, the alphabetical order has been set.

Introduction

The current report involves the third release of our annual series *Scimago Institutions Rankings World Reports*, that based on quantitative data of citation and publication shows bibliometric indicators that unveil some of the main dimensions of research performance of worldwide research-devoted institutions. As in former editions, **SIR World Report 2011** aims at becoming an evaluation framework of research performance to Worldwide Research Organizations.

The report shows six indicators that will help users evaluate the scientific impact, thematic specialization, output size and international collaboration networks of the institutions.

The period analyzed in the current edition covers 2005-09. The tables include institutions having published at least 100 scientific documents of any type, that is, articles, reviews, short reviews, letters, conference papers, etc., during the year 2009 as collected by worldwide leader scientific database Scopus by Elsevier. The report encompasses Higher Education Institutions (HEIs) as well as other research-focused organizations from different sizes, with different missions and from countries in the five continents. Institutions are grouped into five Institutional Sectors: Higher Education, Health System, Government Agencies, Corporations and Others.

To elaborate this report, we carry out the challenging task of identifying and disambiguating all the institutions through an overwhelming number of scientific articles, reviews and conference papers contained in Scopus. The task, which is carried out by a mix of computer and human means, comprises the identification and gathering of institution's affiliation variants under a unique identifiable form as well as the classification into institutional sectors.

SIR World Reports 2011 is the most comprehensive ranking of Worldwide Research Institutions. Following the goal of embracing every institution around the world with significative scientific output, the ranking now includes 3,042 institutions that together are responsible for more than 80% of worldwide scientific output during the term 2005-09 as indexed in Elsevier's Scopus database.

The intended target audience of SIR World Report 2011 is formed by policymakers, research managers, researchers, media and general public interested in finding out about research performance of worldwide Institutions.

Disclaimer notice:

SIR World Report 2011 IS NOT A LEAGUE TABLE. The ranking parameter – the Normalized Impact of institutions-- should be understood as a default rank, not our ranking proposal. The only goal of this report is to characterize research outcomes of organizations so as to provide useful scientometric information to institutions, policymakers and research managers so they are able to analyze, evaluate and improve their research results. If someone uses this report to rank institutions or to build a league table with any purpose, he/she will do it under his/her own responsibility.

Indicators

Selected indicators seek to reveal main aspects of research size, performance, impact and internationalization at Worldwide Research Institutions.

O::Output

An institution's publication output reveals its scientific outcomes in terms of published documents in scholarly journals.

IC::International Collaboration

IC shows an institution's output ratio that has been produced in collaboration with foreign institutions. The values are computed by analyzing the institution's output whose affiliation includes more than one country address over the whole period.

NI::Normalized Impact

The values, expressed in percentages, show the relationship of an institution's average scientific impact and the world average, which is 1, --i.e. a score of 0.8 means the institution is cited 20% below average and 1.3 means the institution is cited 30% above average. More on NI.

Q1::High Quality Publications

Ratio of publications that an institution publishes in the most influential scholarly journals of the world; those ranked in the first quartile (25%) in their categories as ordered by SCImago Journal Rank SJR indicator.

SI::Specialization Index

The Specialization Index indicates the extent of thematic concentration / dispersion of an institution's scientific output. Values range between 0 to 1, indicating generalistic vs. specialized institutions respectively. This indicator is computed according to the Gini Index used in Economy.

ER::Excellence Rate

The Excellence Rate indicates which percentage of an institution's scientific output is included into the set formed by the 10% of the most cited papers in their respective scientific fields. It is a measure of high quality output of research institutions.

Powered by SCOPUS™

WR	RR	CR	Organization	Sector	Country	Region	Output	IC(%)	Q1(%)	NI	Spe	Exc
1	1	1	George Institute for International Health	GO	AUS	OC	362	58.0	82.9	6.17	0.9	30.9
2	1	1	American Cancer Society	HL	USA	NA	600	23.0	83.3	5.94	0.9	37.0
3	2	2	Whitehead Institute for Biomedical Research	GO	USA	NA	759	33.5	95.3	5.72	0.9	64.4
4	3	3	Broad Institute of MIT and Harvard	GO	USA	NA	1,377	49.2	94.1	5.71	0.9	59.6
5	1	1	Wellcome Trust Sanger Institute	HL	GBR	WE	1,581	66.5	90.7	3.98	0.9	49.7
6	4	4	Novartis Pharma SA, East Hanover	CO	USA	NA	932	53.0	74.9	3.46	0.9	42.8
7	5	1	Hamilton Health Sciences	HL	CAN	NA	1,293	36.9	63.7	3.26	0.8	29.6
8	2	1	Institut d'Estudis Espacials de Catalunya	GO	ESP	WE	753	71.1	60.6	3.21	1.0	24.8
9	6	5	Dana Farber Cancer Institute	HL	USA	NA	5,966	30.3	85.9	3.14	0.9	45.5
10	7	6	J. Craig Venter Institute	HL	USA	NA	754	49.6	90.1	3.13	0.9	54.6
11	8	7	Centocor, Incorporated	CO	USA	NA	555	36.4	80.9	3.12	0.9	41.1
12	9	8	Howard Hughes Medical Institute	HL	USA	NA	10,807	30.7	94.9	3.10	0.9	61.7
13	3	2	Microsoft Research Cambridge	CO	GBR	WE	748	58.0	34.2	3.10	0.9	15.8
14	10	9	Kaiser Permanente	HL	USA	NA	1,008	12.5	87.2	3.08	0.9	41.7
15	11	10	F. Hoffmann-La Roche, Ltd.	CO	USA	NA	2,476	31.9	83.1	3.07	0.8	46.7
16	12	11	Institute for Systems Biology	HL	USA	NA	591	56.9	88.3	3.07	0.9	52.1
17	13	2	Institute for Clinical Evaluative Sciences	HL	CAN	NA	786	23.2	71.1	3.01	0.9	31.0
18	14	12	Cold Spring Harbor Laboratory	HL	USA	NA	1,018	42.0	93.0	3.00	0.9	59.4
19	4	1	World Health Organization Switzerland	HL	CHE	WE	2,885	76.6	74.6	2.94	0.9	32.3
20	15	13	Harvard-MIT Division of Health Sciences and Tecnology	HE	USA	NA	607	29.7	72.8	2.91	0.9	35.6
21	5	1	Steno Diabetes Center	HL	DNK	WE	560	51.3	82.5	2.91	1.0	41.3
22	16	3	Institut Universitaire de Cardiologie et de Pneumologie de Quebec	HL	CAN	NA	854	30.7	71.0	2.89	0.9	25.8
23	17	14	New England Research Institutes	GO	USA	NA	454	24.5	90.3	2.89	0.9	39.0
24	6	2	IBM Zurich Research Laboratory	CO	CHE	WE	786	60.4	49.2	2.80	0.9	18.5
25	18	4	University of Alberta Hospital	HL	CAN	NA	549	27.3	63.0	2.80	0.9	21.7
26	19	15	AstraZeneca Pharmaceuticals, LP	CO	USA	NA	667	39.3	79.9	2.79	0.9	35.5
27	20	16	Harvard Pilgrim Health Care	GO	USA	NA	514	14.4	86.6	2.79	0.9	39.3
28	2	1	Auckland City Hospital	HL	NZL	OC	1,067	37.8	67.6	2.77	0.9	20.2
29	7	2	Herlev Hospital	HL	DNK	WE	1,028	34.2	65.1	2.77	0.9	30.5
30	21	17	Centers for Disease Control and Prevention Estados Unidos	HL	USA	NA	13,098	25.3	78.2	2.76	0.8	32.0
31	22	18	Group Health Cooperative	HL	USA	NA	923	13.2	84.8	2.76	0.9	34.8
32	8	1	International Agency for Research on Cancer	HL	FRA	WE	1,512	87.0	86.6	2.74	0.9	43.0
33	9	2	Institut Catala d'Oncologia, Hospitalet de Llobregat	HL	ESP	WE	953	56.8	72.3	2.72	0.9	37.6
34	10	3	AstraZeneca	CO	GBR	WE	598	47.3	75.8	2.69	0.8	36.1
35	23	19	Northshore University HealthSystem	HL	USA	NA	805	18.5	76.2	2.69	0.9	31.7
36	24	20	California Pacific Medical Center	HL	USA	NA	726	21.9	81.3	2.66	0.9	36.9
37	11	1	European Molecular Biology Laboratory Heidelberg	GO	DEU	WE	1,447	63.4	92.2	2.62	0.9	52.3
38	12	1	Jules Bordet Institute	HL	BEL	WE	607	45.3	65.1	2.62	0.9	26.5
39	25	21	Partners HealthCare System	HL	USA	NA	38,096	28.5	80.7	2.62	0.7	36.5
40	13	1	Landspitali National University Hospital	HL	ISL	WE	744	62.6	68.0	2.61	0.8	29.4
41	26	5	Perimeter Institute for Theoretical Physics	OT	CAN	NA	810	68.8	56.4	2.60	1.0	30.3
42	27	22	Armed Forces Institute of Pathology	GO	USA	NA	709	31.6	77.9	2.58	0.9	33.9
43	14	1	Fondazione IRCCS Istituto Nazionale Tumori di Milano	HL	ITA	WE	1,844	42.5	79.8	2.58	0.9	35.8
44	15	4	Institute of Cancer Research	HL	GBR	WE	2,100	42.0	83.4	2.58	0.9	44.1
45	28	23	Bristol-Myers Squibb Company	CO	USA	NA	1,581	25.1	76.6	2.57	0.9	36.9
46	29	24	Institute for Advanced Study	HE	USA	NA	1,282	45.9	60.9	2.57	0.9	30.9
47	30	25	Microsoft Corporation	CO	USA	NA	3,004	32.2	38.7	2.57	0.9	13.8
48	31	26	Pennington Biomedical Research Center	HL	USA	NA	886	28.3	80.7	2.57	0.9	38.4
49	32	27	Salk Institute for Biological Studies	HL	USA	NA	1,332	43.8	92.9	2.57	0.9	58.8
50	16	5	London School of Hygiene and Tropical Medicine	HE	GBR	WE	5,831	63.7	82.7	2.56	0.8	32.3
51	33	6	British Columbia Cancer Agency	HL	CAN	NA	1,543	45.9	80.0	2.53	0.9	39.5
52	34	28	Fred Hutchinson Cancer Research Center	HL	USA	NA	4,776	28.6	88.9	2.53	0.9	42.7
53	35	29	GlaxoSmithKline, United States	CO	USA	NA	2,817	35.9	80.1	2.53	0.8	36.4
54	36	7	IBM Research	CO	CAN	NA	959	99.8	55.6	2.52	0.9	21.0
55	17	3	Institut Catala d'Investigacio Quimica	GO	ESP	WE	563	46.0	84.4	2.52	1.0	45.1
56	37	30	Ohio State University Comprehensive Cancer Center	HL	USA	NA	1,070	30.3	87.3	2.52	0.9	43.9
57	38	8	University of Ottawa Heart Institute	HL	CAN	NA	604	32.0	77.0	2.52	0.9	27.3
58	39	31	Eli Lilly and Company	CO	USA	NA	3,013	34.0	77.1	2.51	0.8	37.6
59	40	32	Qualcomm Incorporated	CO	USA	NA	940	24.4	58.5	2.51	1.0	13.1

WR/RR/CR World/Region/Country Rank
AF Africa AS Asia EE Eastern Europe LA Latin America ME Middle East NA Northern America OC Oceania WE Western Europe
GO Government HE Higher Education HL Health PR Private OT Others

Outline

- Intro: recalling past stories
- The accelerating Universe
- Different types of models
- Several problems
- Models with non-local interactions (Deser & Woodard): motivations
- Our model of type $f(\Box^{-1}R)$

With THANKS to:

Sergey Yu. Vernov, Ying-li Zhang, Ekaterina Pozdeeva, Sergei Odintsov, Misao Sasaki, Guido Cognola, Sergio Zerbini


12+
SMITHSONIAN
WASHINGTON
26 APRIL 1920
VS
HARLOW
SHAPLEY
HEBER
CURTIS
GREAT DEBATE
МИЦ
WORLD HEAVYWEIGHT CHAMPIONSHIP
WBA / IBF / WBO / IBO
MOSCOW 2013
РОСНЕФТЬ

An Island Universe

- Apr 26, 1920: The Great Debate – Shapley vs Curtis

  Harlow Shapley – the Milky Way was the entire Universe

  Heber Curtis – many novae in Andromeda: “island Universe" (I Kant)

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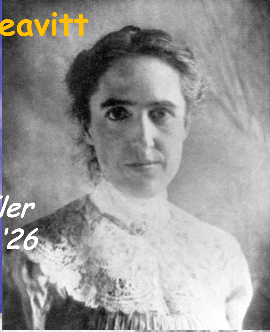
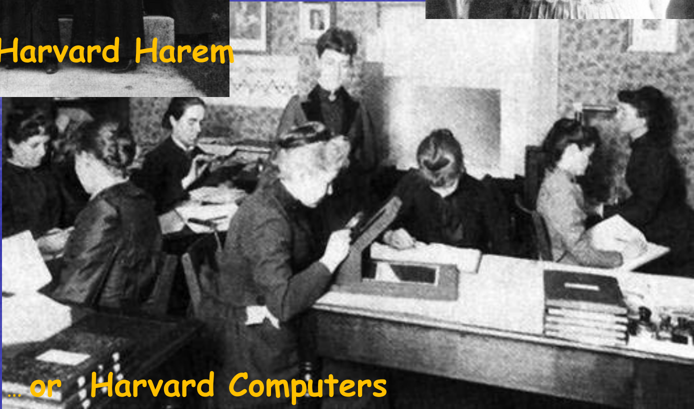
Ed Pickering's Harvard Harem

1913

Henrietta Leavitt

Mittag-Leffler
NP proposal '26





... or Harvard Computers

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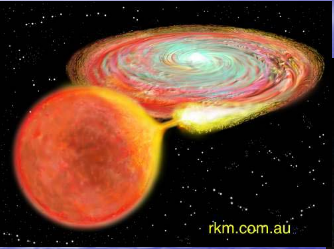
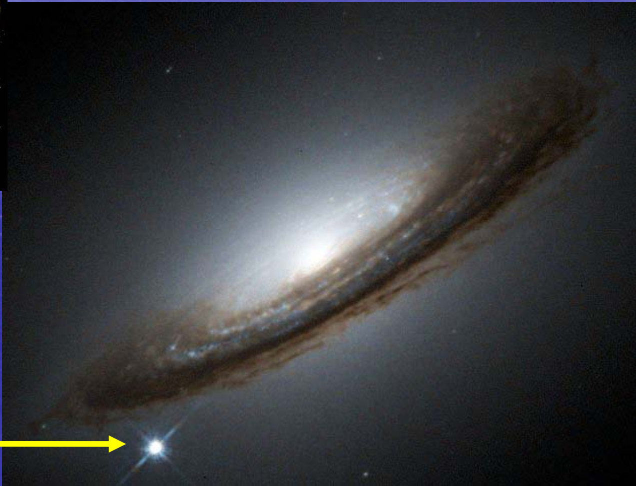
- Present “island Universe": Josiah McElheny, glass artist, White Cube London, Nat Mus Reina Sofia Madrid – The Multiverse Feeney ea ’11

- Karl Schwarzschild: Black Hole solution (22 Dec 1915)
- Willem de Sitter: massless univer expand sol (just cc,'17)
- Alexander Friedmann: expanding universe sol (1922)
- Georges Lemaître: expanding universe (MIT 1925, AF solution); visited Vesto Slipher (Lowell Obs, Arizona, 1912 redshifts) and Edwin Hubble (Mount Wilson, Pasadena); Keeler-Slipher-Campbell 1918, redshifts
- At meeting in Brussels, 1927, Lemaître said to Einstein: *Universe is expanding, no need for* λ. Answer: *no error but…* It took Einstein over two years to understand. Then he pronounced his very famous sentence: *Weg mit*
- Looking backwards in time: The Universe had an origin! Primeval atom (Nature '31). The Church was happy with Lemaître's scenario: Monsignor and later President of the Pontifical Academy of Sciences *(not inspired by Genesis)*

Two groups looked for the 'deceleration' of the universe expansion, using type Ia supernovae as standardizable candles



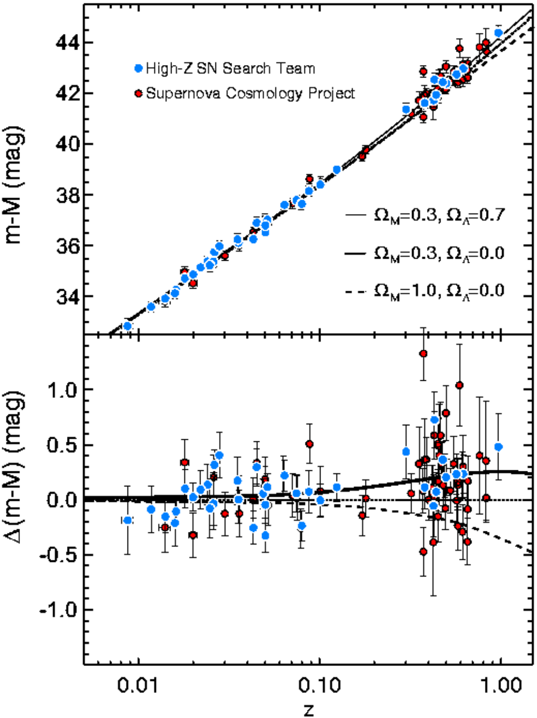


SN 1994d

Result: supernovae are dimmer than expected

*The universe is **not** decelerating*
*It is **accelerating***

Cannot be explained by matter+radiation (see before)



Riess, Schmidt et al. '98
Perlmutter et al. '99

The accelerating Universe

Dark Energy: effects on the expansion rate of Universe, 3 approaches:

—- Standard candles: measure luminosity distance as function of redshift

—- Standard rulers: angular diam distance & expansion rate as f of redsh

—- Growth of fluctuations: generated at origin of U & amplified by inflation

Both angular diameter and luminosity distances are integrals of (inverse) expansion rate: encode effects of DE

To get competitive constraints on DE need see changes in $H(z)$ at 1% would give statistical errors in DE EoS of $O(10\%)$

—- calibrate the ruler accurately over most of the age of the universe

—- measure the ruler over much of the volume of the universe

—- make ultra-precise measurements of the ruler

On large scales or early times the perturbative treatment is valid: calculations are perfectly under control

Length scales from physics of the early universe are imprinted on the distribution of mass and radiation: they form time-independent rulers

(M White, Berkeley)

The accelerating Universe (II)

Evidence for the acceleration of the Universe expansion:

- distant supernovae

  A.G Riess et al. ’98, S. Perlmutter et al. ’99

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- baryon acoustic oscillations BAO Martin White *mwhite/bao/*

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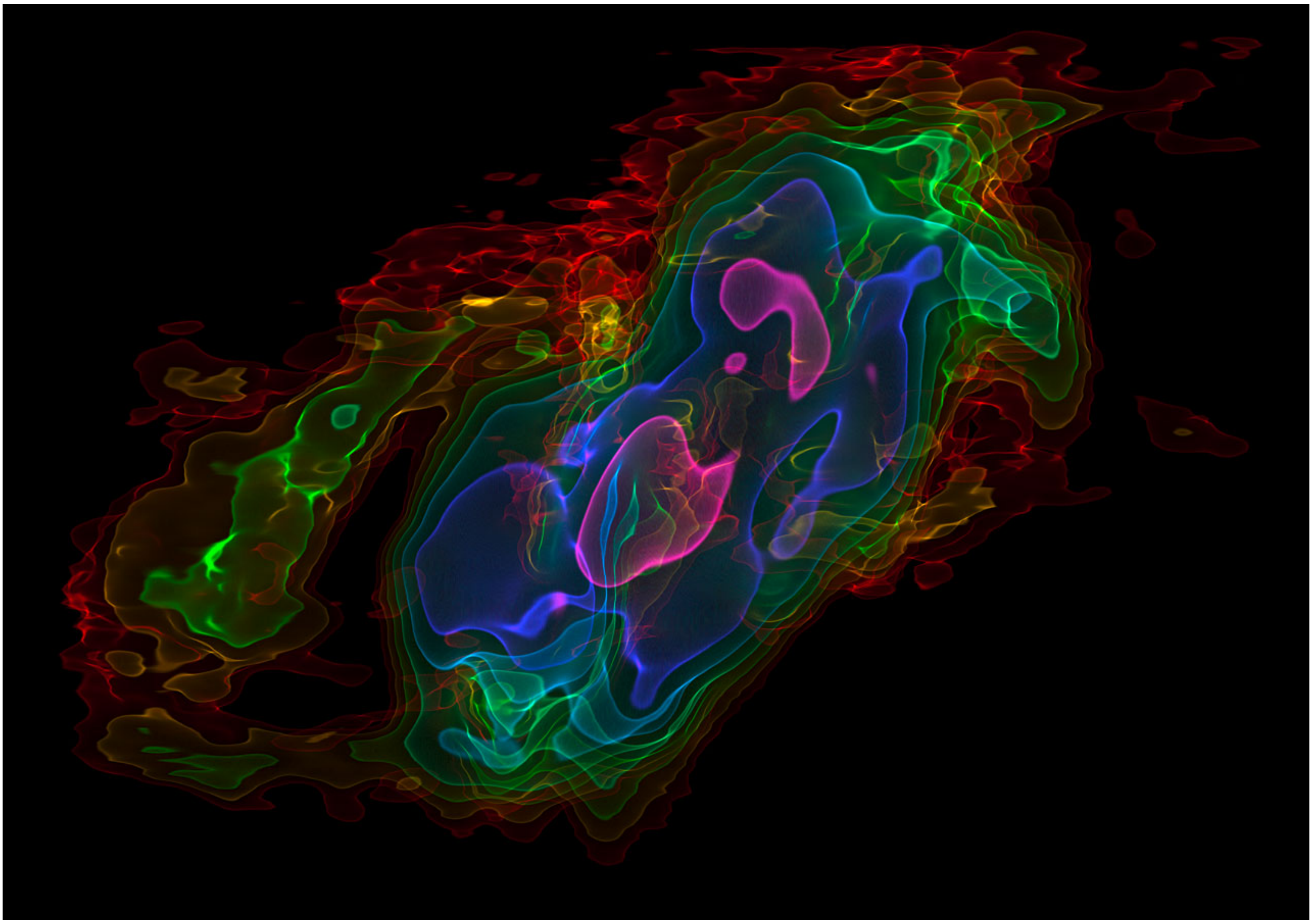
- the galaxy distribution AG Sanchez ea 12, Gaztanaga ea

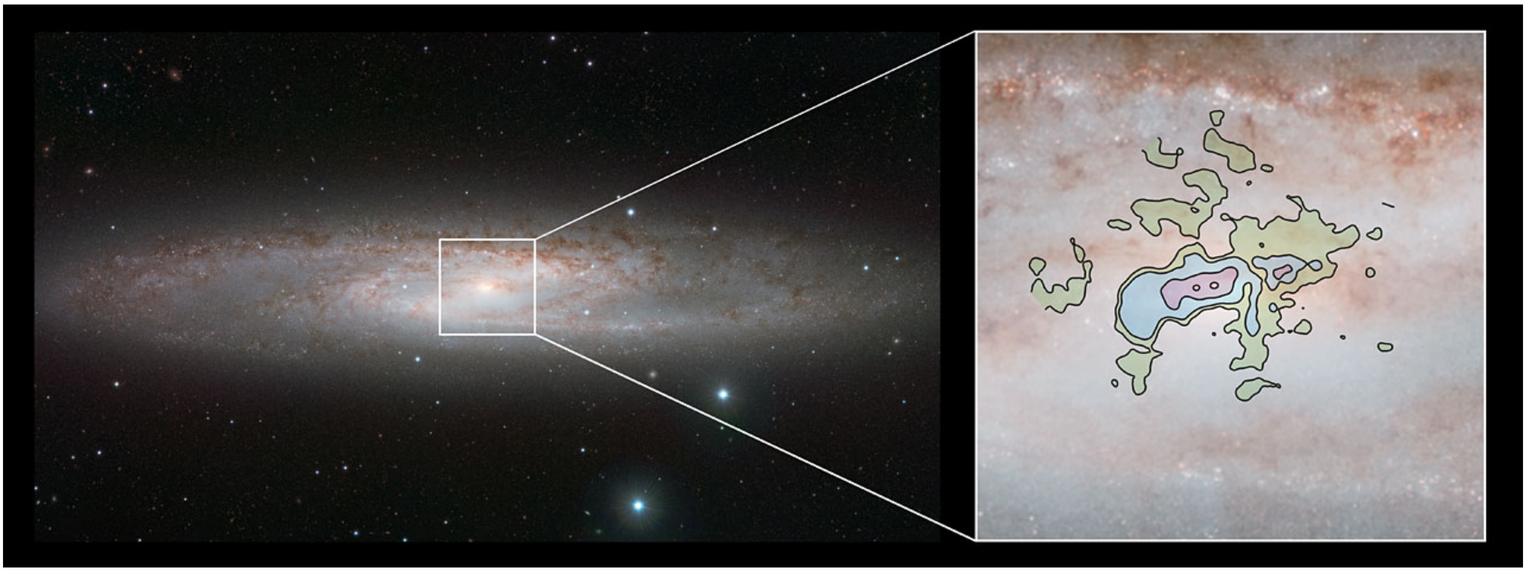
- correlations of galaxy distribs R Scranton ea ’03,

Multiple sets of evidence: no systematics affect the conclusion that $\ddot{a} > 0$, a scale factor of the Universe

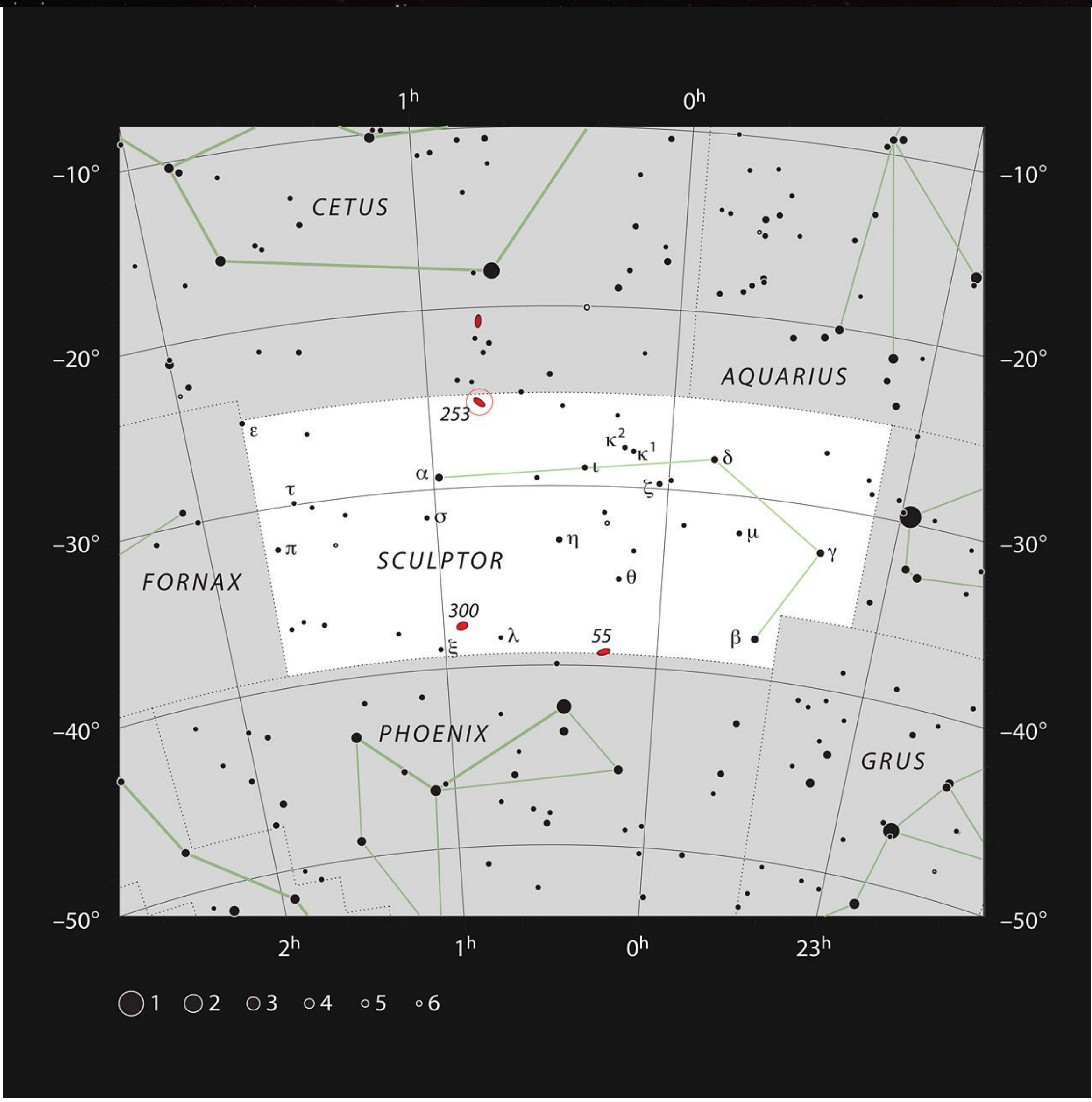
Starburst to Star Bust

ALMA on Mystery of Missing Massive Galaxies **Nature25-07-13 A Bolatto ea**





Vigorous star formation can blast gas out of a galaxy and starve future generations of stars of the fuel they need to form and grow. Enormous outflows of molecular gas are ejected by star-forming regions in the nearby Sculptor Galaxy NGC253 (11.5MLY) They help explain the strange paucity of very massive galaxies in the Universe.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−50°
CETUS
AQUARIUS
SCULPTOR
FORNAX
PHOENIX
GRUS
253
300
55
α
β
γ
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ι
κ2
κ1
λ
μ
ξ
π
σ
τ
2h
1h
0h
23h
1
2
3
4
5
6

Different types of models

- In General Relativity (GR): Gravity leads to deceleration

  But pressure also influences geometry: R Tolman '32

  negative pressure can drive acceleration

  Cosmological evidence could be explained by an undiscovered substance with negative pressure, so-called dark energy

  J.A. Frieman ea '95, K. Coble ea '97, R. Caldwell ea '97, B. Ratra, P.J.E. Peebles '98, C. Wetterich '98, D. Huterer, M.S. Turner '99

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- Another possib: GR is (wrong!) not accurate enough at large scales

  S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner '04, Capozziello '04

  GR was developed using information and intuition at Solar System scales, it is (almost) only checked there, it could need to be modified on large scales

  A. Starobinsky arrived at same conclusion, through very different arguments based on quantum corrections to ordinary GR: lead to terms of second order in R, and higher

Several problems

- Do not have simple guidelines, gedanken experiment, reasons of elegance and simplicity, as those of Einstein in constructing GR

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- Another problem: modified gravity models should comply with the successes of GR in the Solar System

  These constraints already doomed one of the first most promising modified gravity models introduced to explain acceleration and still place tight constraints on many models

Models with non-local interactions

- One class of modified gravity models that overcomes most of these problems contains non-local interactions

  S. Deser and R. Woodard, Phys.Rev.Lett. 99, 111301 (2007), 0706.2151

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- At cosmological scales, $\Box^{-1}R$ grows very slowly:
  - as $(t/teq)^{1/2}$ in the radiation dominated era
  - logarithmically in the matter dominated era
  - So, at the time of Nucleosynthesis $\Box^{-1}R$ is about 10^{-6} and at matter-radiation equilibrium it is only order 1

  In a natural way, these terms are irrelevant at early times and begin to affect the dynamics of the Universe only after the matter-radiation transition
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- Since $\Box^{-1}R$ is dimless, the functional $\times R$ has no new mass parameter

- Finally, because $\Box^{-1}R$ is extremely small in the Solar System, these models easily pass local tests of gravity

Sufficient theoretical motivation

- In string theory, $R\,\Box^{-1}R$ is precisely the term generated by quantum corrections (away from the critical dimension) as first pointed out by Polyakov

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- A realistic model for acceleration, with an arbitrary function of $R\,\Box^{-1}R$, reproducing the expansion history of ΛCDM

  C. Deffayet, R. Woodard, JCAP 0908, 023 (2009), 0904.0961

  Used by S. Park & S. Dodelson, arXiv:1209.0836, to discuss structure formation in a nonlocally modified gravity

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- Anyway, the very first problem still remains: a completely arbitrary free function can be chosen at will to fit the expansion history
- But a very interesting aspect of modified gravity models in general is that, even if they are constructed to reproduce the expansion history equivalent to that of a given dark energy model (such as ΛCDM), perturbations will often evolve differently than in a model with standard GR plus dark energy
- Indeed, the way to distinguish DE models from modified gravity is to measure the growth of structure in the Universe

  The deviations from dark energy models are at the 10 to 30$\%$ level and have a characteristic signature as a function of redshift, which suggests that the class of models could be tested by upcoming surveys

Our model of type $f(\Box^{-1}R)$

- Consider a nonlocal gravity which contains a function of the $\Box^{-1}$ operator, thus not assuming the existence of a new dimensional parameter in the action. We focus on the study of cosmological solutions both in Jordan and Einstein frames, including matter in the last case

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- Several models are able to reproduce observations, as quintom models, which involve two fields: a phantom and an ordinary scalar

  E Elizalde, S Nojiri, S Odintsov, Phys.Rev.D70(2004)043539, hep-th/0405034

  W Zhao and Y Zhang, Phys.Rev. D73 (2006) 123509, arXiv:astro-ph/0604460

  H Štefančić, Phys.Rev. D71(2005)124036, arXiv:astro-ph/0504518

Our model (cont.)

- The nonlocal model we will consider is usually studied in the Jordan frame, but recently its behavior in the Einstein frame, for the case without matter, has been explored too

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- We analyze the correspondence between solutions got in different frames and prove explicitly how knowledge of power-law solutions in Jordan's frame can be used to get power-law solutions in Einstein's one

The action

- Consider a class of nonlocal gravities, with action

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[R\left(1+f(\Box^{-1}R)\right)-2\Lambda\right]+\mathcal{L}_{\rm m}\right\}$$

where $\kappa^2 = 8\pi G = 8\pi/{M_{\rm Pl}}^2$, the Planck mass being $M_{\rm Pl} = G^{-1/2} = 1.2\times 10^{19}$ GeV, f differentiable function (characterizes nature of nonlocality), $\Box^{-1}$ inverse of d'Alembertian operator, Λ cosmological constant, and $\mathcal{L}_{\rm m}$ matter Lagrangian. For definiteness, we assume that matter is a perfect fluid. We use the signature $(-,+,+,+)$, g determinant of $g_{\mu\nu}$

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- Introducing two scalar fields: $\psi = \Box^{-1} R$ & Lagrange multiplier ξ

$$S_{loc} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} \left[R \left(1 + f(\psi) \right) + \xi \left(R - \Box\psi \right) - 2\Lambda \right] + \mathcal{L}_{\rm m} \right\}$$

  Original non-local action is recast as a local action in the Jordan frame. Varying this action with respect to ξ and ψ, one resp gets the field eqs

$$\Box\psi = R \,, \qquad \Box\xi = f_{,\psi}(\psi) R \,,$$

  where $f_{,\psi}(\psi) \equiv {\rm d}f/{\rm d}\psi$

The action (II)

- The corresponding Einstein equations are obtained by variation of the local action wrt the metric tensor

$$\frac{g_{\mu\nu}}{2}\left[R\Psi + \partial_\rho\xi\partial^\rho\psi - 2(\Lambda + \Box\Psi)\right] - R_{\mu\nu}\Psi - \frac{1}{2}\left(\partial_\mu\xi\partial_\nu\psi + \partial_\mu\psi\partial_\nu\xi\right) + \nabla_\mu\partial_\nu\Psi = -\kappa^2 T$$

where $\Psi \equiv 1 + f(\psi) + \xi$, and $T_{(\mathrm{m})\,\mu\nu}$ energy-momentum tensor of matter sector

$$T_{(\mathrm{m})\,\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_{\mathrm{m}}\right)}{\delta g^{\mu\nu}}$$

Note the system of equations here does not include the function ψ itself, but instead $f(\psi)$ and $f_{,\psi}(\psi)$, together with time derivatives of ψ. Also, $f(\psi)$ can only be determined up to a constant: one may add a constant to $f(\psi)$ and subtract the same constant from ξ without changing eqs

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- Here, we assume a spatially flat FLRW universe

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

  and consider the case where the scalar fields $\psi(t)$ and $\xi(t)$ are functions of the cosmological time t only

The action (III)

- Thus, the system of Eqs. reduces to

$$\begin{aligned} 3H^2\Psi &= -\frac{1}{2}\dot{\xi}\dot{\psi} - 3H\dot{\Psi} + \Lambda + \kappa^2\rho_{\mathrm{m}} \\ \left(2\dot{H} + 3H^2\right)\Psi &= \frac{1}{2}\dot{\xi}\dot{\psi} - \ddot{\Psi} - 2H\dot{\Psi} + \Lambda - \kappa^2 P_{\mathrm{m}} \\ \ddot{\psi} &= -3H\dot{\psi} - 6\left(\dot{H} + 2H^2\right) \\ \ddot{\xi} &= -3H\dot{\xi} - 6\left(\dot{H} + 2H^2\right) f_{,\psi}(\psi) \end{aligned}$$

dot means differentiation with respect to time, t, in the Jordan frame: $\dot{A}(t) \equiv \mathrm{d}A(t)/\mathrm{d}t$, $H = \dot{a}/a$ Hubble parameter

For a perfect matter fluid, $T_{(\mathrm{m})\,00} = \rho_{\mathrm{m}}$ and $T_{(\mathrm{m})\,ij} = P_{\mathrm{m}} g_{ij}$

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- The continuity equation is

$$\dot{\rho}_{\mathrm{m}} = -3H(P_{\mathrm{m}} + \rho_{\mathrm{m}})$$

Adding them we obtain a 2nd order linear differential equation for Ψ

$$\ddot{\Psi} + 5H\dot{\Psi} + \left(2\dot{H} + 6H^2\right)\Psi - 2\Lambda + \kappa^2(P_{\mathrm{m}} - \rho_{\mathrm{m}}) = 0$$

Power-law sol's with $f(\psi)$ exp funct

- Consider the case when $f(\psi)$ exponential function

$$f(\psi) = f_0 e^{\alpha\psi}$$

f_0 and α nonzero real parameters. The motivation: (i) simplest model with power-law and de Sitter solutions (only exp or a sum of exps); (ii) better studied case among all possible functions for expanding universe sol's (with Hubble parameter $H = n/t$, with n a nonzero constant)

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- We consider matter with the EoS parameter $w_{\mathrm{m}} \equiv P_{\mathrm{m}}/\rho_{\mathrm{m}}$ being a constant but not equal to -1. For power-law solutions $H = n/t$, the continuity eq has the following general solution (ρ_0 arbitrary const)

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- Solutions with $H = n/t$. Inserting $H = n/t$

$$\psi(t) = \psi_1 t^{1-3n} - \frac{6n(2n-1)}{3n-1} \ln\left(\frac{t}{t_0}\right)$$

ψ_1, t_0 integration const's. We consider real solutions at $t > 0$, hence, $t_0 > 0$. Note these valid provided $n \neq 1/3$ and $n \neq 1/2$ (special cases, other sect)

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  - Newtonian limit of the theory, described by corresponding local action check whether the power-law solutions found can satisfy this constraint
  - Depending on the integration constant ξ_1 being vanishing or not, we draw very different constraints on the Post-Newtonian parameter γ:
    - $\xi_1 \neq 0$, the constraint can be easily satisfied for a wide range of choices of the parameter α
    - $\xi_1 = 0$ one needs to tune the parameter α to at least 10^{-5} order, to satisfy the local constraint

  Note that, in previous papers, $\xi_1 = 0$ for simplicity. Analysis of the local constraint shows that solutions with nonzero ξ_1 allow to change the restrictions on the parameter α, which are indeed necessary in order to make the model compatible with astronomical observations
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- Varying the nonlocal action wrt the metric $g_{\mu\nu}$, under the spatially flat FLRW metric, the independent components of the field equations are

$$3H^2 + \Delta G_{00} = \kappa^2 \rho_{\mathrm{m}} + \Lambda, \qquad -2\dot{H} - 3H^2 + \frac{1}{3a^2}\delta^{ij}\Delta G_{ij} = \kappa^2 P_{\mathrm{m}} - \Lambda$$

ΔG_{00} and ΔG_{ij} are the modifications coming from the nonlocal terms

$$\begin{aligned}
\Delta G_{00} &= \left[3H^2 + 3H\partial_t\right]\left\{f\left(\Box^{-1}R\right) + \Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right\} \\
&+ \frac{1}{2}\partial_t\left(\Box^{-1}R\right)\partial_t\left(\Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right), \\
\Delta G_{ij} &= a^2\delta_{ij}\left[\frac{1}{2}\partial_t\left(\Box^{-1}R\right)\partial_t\left(\Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right)\right. \\
&- \left.\left[2\dot{H} + 3H^2 + 2H\partial_t + \partial_t^2\right]\left\{f\left(\Box^{-1}R\right) + \Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right\}\right]
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&+ \frac{1}{2}\partial_t\left(\Box^{-1}R\right)\partial_t\left(\Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right), \\
\Delta G_{ij} &= a^2\delta_{ij}\left[\frac{1}{2}\partial_t\left(\Box^{-1}R\right)\partial_t\left(\Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right)\right. \\
&- \left.\left[2\dot{H} + 3H^2 + 2H\partial_t + \partial_t^2\right]\left\{f\left(\Box^{-1}R\right) + \Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right\}\right]
\end{aligned}$$

- Identifying the scalar fields ψ and ξ with corresponding terms in original action

$$\psi(t) = \Box^{-1}R, \qquad \xi(t) = \Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]$$

These fields $\psi(t)$ and $\xi(t)$ satisfy indeed the local equations

- Varying the nonlocal action wrt the metric $g_{\mu\nu}$, under the spatially flat FLRW metric, the independent components of the field equations are

$$3H^2 + \Delta G_{00} = \kappa^2 \rho_{\mathrm{m}} + \Lambda, \qquad -2\dot{H} - 3H^2 + \frac{1}{3a^2}\delta^{ij}\Delta G_{ij} = \kappa^2 P_{\mathrm{m}} - \Lambda$$

ΔG_{00} and ΔG_{ij} are the modifications coming from the nonlocal terms

$$\begin{aligned} \Delta G_{00} &= [3H^2 + 3H\partial_t]\left\{f\left(\Box^{-1}R\right) + \Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right\} \\ &+ \frac{1}{2}\partial_t\left(\Box^{-1}R\right)\partial_t\left(\Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right), \\ \Delta G_{ij} &= a^2\delta_{ij}\left[\frac{1}{2}\partial_t\left(\Box^{-1}R\right)\partial_t\left(\Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right)\right. \\ &- \left.\left[2\dot{H} + 3H^2 + 2H\partial_t + \partial_t^2\right]\left\{f\left(\Box^{-1}R\right) + \Box^{-1}\left[R\frac{df}{d(\Box^{-1}R)}\right]\right\}\right] \end{aligned}$$

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- We conclude that these solutions are solutions of the initial nonlocal model as well, what can be checked immediately by direct substitution

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  In fact, if we write $\psi \longrightarrow \psi + \chi$ the corresponding term is

$$\xi(\Box\psi - R) \longrightarrow \xi\big(\Box(\psi + \chi) - R\big)$$

  after integration by parts, change is $g^{\mu\nu}\partial_\mu \xi \partial_\nu \psi \longrightarrow g^{\mu\nu}\partial_\mu \xi \partial_\nu (\psi + \chi)$

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- However, imposing appropriate BC, as $\chi = 0$, to recover original form, this would-be extra dof can be eliminated. Issue of choice of correct BC should be the only non-equivalence between original form and biscalar-tensor representation. Thus, eg, Woodard ea determine the inverse d'Alembert op using the retarded Green function: they fix a solution of eq $\Box R = 0$ putting $\widetilde{t}_0 = 0$ and $\eta_0 = 0$

- A final comment. As stated above, the biscalar-tensor representation introduces two scalars, ψ and ξ, therefore, working in this way it seems that one will encounter a ghost-like behavior (Koivisto, Nojiri, Bamba, Sasaki) However, since the original nonlocal model does not introduce any new degree of freedom, the ghost-like behavior of the biscalar-tensor theory may not be physically relevant: associated terms can be cast as a boundary term of the nonlocal operators (Koivisto 2010). At classical level, a necessary way to check this physical relevance is by considering the equivalence of the solutions coming from the original nonlocal formulation and from its biscalar-tensor form

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- Action and equations of motion in the Einstein frame
- The Jordan and Einstein frames

  Once a modified gravity theory is recast into its scalar-tensor presentation, it immediately follows that both the Jordan frame (where the matter sector minimally couples to gravity) and the Einstein one (where the Ricci is linear but matter couples to gravity non-minimally) are available

  They are related by conformal transformation $g_{\mu\nu} = \Omega^2 g^{(E)}_{\mu\nu}$

  metric in Jordan frame is $g_{\mu\nu}$, in Einstein frame, $g^{(E)}_{\mu\nu}$

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- First, formulate diff eqn for the conformal factor under which power-law sol’s in Jordan’s correspond to other power-law sol’s in Einstein’s

Conclusions

- In GR, power-law solutions of type $H = n/t$ correspond to models with a perfect fluid whose EoS param $w_{\mathrm{m}} \equiv P_{\mathrm{m}}/\rho_{\mathrm{m}}$ is related to power-index by $w_{\mathrm{m}} = -1 + 2/(3n)$

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- Recasting original nonlocal action into local form, we obtained power-law sol's, with and without a cc, both in Jordan and Einstein frames. Showed that power-law sol's got in Jordan frame satisfy the original nonlocal equations. In the Jordan frame we proved that all power-law solutions are found that way

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- In Einstein's, we obtained the power-law solutions either by solving the EoM, or by performing a conformal transformation of the sol's obtained in Jordan's. For this purpose, we extended the correspondence to include the matter sector. Using this powerful, non-trivial tool, we got the sol's when $w_{\rm m} = 1/3$ and $w_{\rm m} = 1$ (very difficult to obtain by directly solving the system), thus proving the usefulness of the method

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- Theories with higher derivatives often suffer from a ghost problem, namely a wrong sign in the kinetic term, resulting in dangerous instabilities. Good aspect of conformal transformation technique between frames: obtain corresponding ghost-free conditions
- The biscalar-tensor representation introduces two extra scalars. Can lead to a ghost problem. Equivalence between the initial nonlocal theory and local formulation has not been established yet The ghost-like behavior of the biscalar-tensor theory may not be a physical problem, since the associated terms can be cast as boundary terms of the nonlocal operators (would-be ghost mode might correspond to an inappropriate choice of BC). Plan to consider this important question in future work

- An analysis of the stability of the solutions here encountered, of the ghost-free conditions, and of the cosmological perturbations corresponding to these models will be carried out next

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- E. Elizalde, E.O. Pozdeeva, S.Yu. Vernov, Y.-l. Zhang
  Cosmological solutions of a nonlocal model with a perfect fluid
  J. Cosm & Astrop Phys JCAP07, 034 (2013);
  arXiv:1302.4330[hep-th]

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