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Casimir Momentum of a Chiral Molecule in a Magnetic Field

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Outline

I- Introduction to Casimir momentum

II- Semi-classical approach

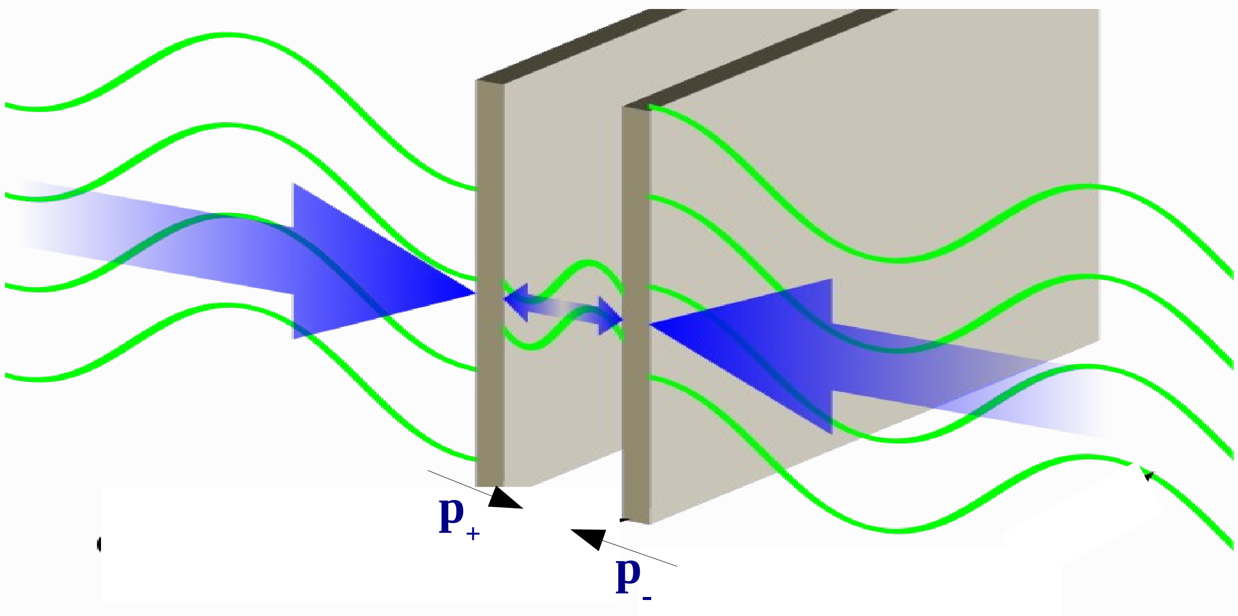
III-Quantum approach

IV- Conclusions and prospective work

I- Introduction to Casimir momentum

<u>.Casimir momentum</u>: Net momentum transferred from the quantum vacuum to a material object.

.It differs from momentum transferred to metallic plates in usual Casimir effect

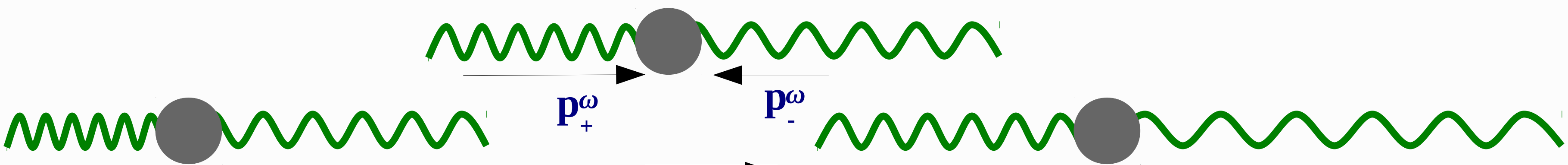


Δp=0

I- Introduction to Casimir momentum

.*Casimir momentum*: Net momentum transferred from the quantum vacuum to a material object.

.*Spectral non-reciprocity* is needed: $\omega(\mathbf{k}) \neq \omega(-\mathbf{k})$ or $n_{+\hat{\mathbf{k}}}(\omega) \neq -n_{-\hat{\mathbf{k}}}(\omega)$



$$\overrightarrow{\Delta \mathrm{p}_{\omega}} \neq \mathbf{0}$$

$$\Delta \mathbf{p}_{\omega} = \hat{\mathbf{k}}\hbar(\omega/c)[n_{+\hat{\mathbf{k}}}(\omega) + n_{-\hat{\mathbf{k}}}(\omega)]$$

I- Introduction to Casimir momentum

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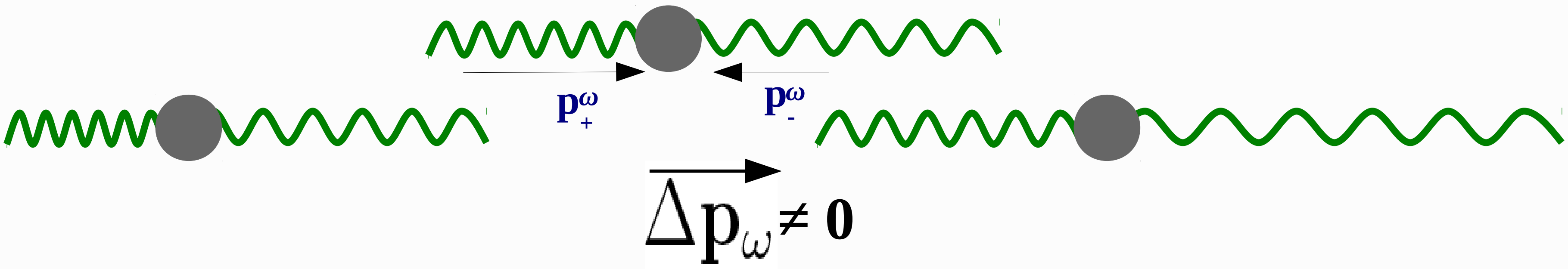

$$\Delta \mathbf{p}_{\omega} = \hat{\mathbf{k}}\hbar(\omega/c)[n_{+\hat{\mathbf{k}}}(\omega) + n_{-\hat{\mathbf{k}}}(\omega)]$$

.Breakdown of time-reversal (T) & parity (P) symmetries is required.

I- Introduction to Casimir momentum

.Non-reciprocity (P & T violation) meets in magneto-electric media,

$$\begin{aligned} \mathbf{D}_\omega &= \bar{\varepsilon}\mathbf{E}_\omega + (\bar{\gamma} + i\omega\bar{\beta})\mathbf{H}_\omega \\ \mathbf{B}_\omega &= \bar{\mu}\mathbf{H}_\omega + (\bar{\gamma}^t - i\omega\bar{\beta}^t)\mathbf{E}_\omega \end{aligned}$$

.Reciprocity is broken in a given direction by $\bar{\gamma} \neq 0$, with $\bar{\gamma}$ an antisymmetric T,P-odd tensor,

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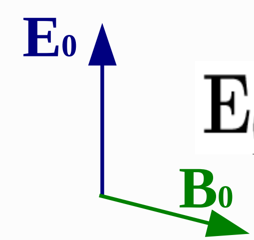
$$\gamma_{ij} \sim \mathrm{E}_i^0\mathrm{B}_j^0 - \mathrm{B}_i^0\mathrm{E}_j^0$$

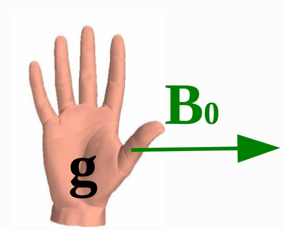
magneto-electric medium
in crossed external fields



$$\mathrm{E}_0 \wedge \mathrm{B}_0$$

$$\gamma_{ij} \sim g\varepsilon_{ijk}\mathrm{B}_0^k$$

chiral object in
a magnetic field



II- Semi-classical approach

Feigel's approach

A.Feigel, Phys.Rev.Lett. (2004)

He considered a homogeneous magneto-electric liquid (in crossed fields $\mathbf{E}_0 \perp \mathbf{B}_0$).

$$\mathcal{L}_{MF} = (8\pi)^{-1} \int \mathrm{d}^3 r (\mathbf{E} \cdot \mathbf{D} - \mathbf{B} \cdot \mathbf{H})$$

.With the liquid at rest,

$$\mathcal{L}^0_{MF} = (8\pi)^{-1} \int \mathrm{d}^3 r [\epsilon \mathbf{E}^2 - \mathbf{B}^2/\mu + 2\mathbf{B} \cdot (\bar{\gamma}^t \mathbf{E})/\mu] + \mathcal{O}(\gamma^2).$$

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.With the liquid in motion at velocity $\boldsymbol{v}$ the fields transform as,

$$\mathbf{E} \to \mathbf{E} + c^{-1} \mathbf{v} \wedge \mathbf{B}$$

$$\mathbf{B} \to \mathbf{B} - c^{-1} \mathbf{v} \wedge \mathbf{E}$$

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.With the liquid in motion at velocity ***v***,

$$\mathcal{L}^{v}_{MF} = \mathcal{L}^{0}_{MF} + \int \mathrm{d}^3 r \{\rho \mathbf{v}^2/2 + \frac{\mathbf{v}\cdot}{4\pi\mu c}[(\epsilon\mu - 1)\mathbf{E}\wedge\mathbf{B} + \mathbf{E}\wedge\bar{\gamma}^t\mathbf{E} + \bar{\gamma}\mathbf{B}\wedge\mathbf{B}]\}$$

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Equation of motion for the liquid reads,

$$\rho \mathbf{v} = \frac{1}{4\pi\mu c}[(\epsilon\mu - 1)\mathbf{E} \wedge \mathbf{B} + \mathbf{E} \wedge \bar{\gamma}^t \mathbf{E} + \bar{\gamma}\mathbf{B} \wedge \mathbf{B}],$$

which is quadratic in the fields.

II- Semi-classical approach

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.Fields are interpreted as quantum operators acting on the EM vacuum,

$$\rho\mathbf{v} = \frac{1}{4\pi\mu c}[(\epsilon\mu - 1)\langle\mathbf{E}\wedge\mathbf{B}\rangle - \bar{\gamma}^t\langle\mathbf{E}\wedge\mathbf{E}\rangle + \bar{\gamma}\langle\mathbf{B}\wedge\mathbf{B}\rangle]$$

.Applying the fluctuation-dissipation theorem,

$$\rho\mathbf{v} = \frac{\hat{\mathbf{k}}\hbar}{3\pi^2 c^4}\int_0^\infty \frac{1+\epsilon\mu}{\mu}\gamma_\perp\omega^3 \mathrm{d}\omega \ ,$$

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which is equivalent to the momentum of 'dressed' radiative modes,

$$\rho\mathbf{v} = \mathcal{V}^{-1}\sum_{(\omega/c)\hat{\mathbf{k}}} \hbar(\omega/c)\hat{\mathbf{k}}[n_{+\hat{\mathbf{k}}}(\omega) + n_{-\hat{\mathbf{k}}}(\omega)] \, .$$

O.A.Croze, Proc.R.Soc. A (2012)

II- Semi-classical approach

Flaws found in the semi-classical approach:

a. Lack of an explicit interaction between matter and radiation. Use of Lorentz force and constitutive equations instead yields different prefactors.

D.F.Nelson, Phys.Rev. A (1991)

B.van Tiggelen, Eur.Phys.J. D (2008)

b. $$\rho\mathbf{v} = \frac{\hat{\mathbf{k}}\hbar}{3\pi^2 c^4}\int_0^\infty \frac{1+\epsilon\mu}{\mu}\gamma_\perp \omega^3 \mathrm{d}\omega$$ is generally UV-divergent.

c. Usage of macroscopic magneto-chiral parameters yields different result to (still semi-classical) microscopic calculation. B.van Tiggelen, Eur.Phys.J. D (2008)

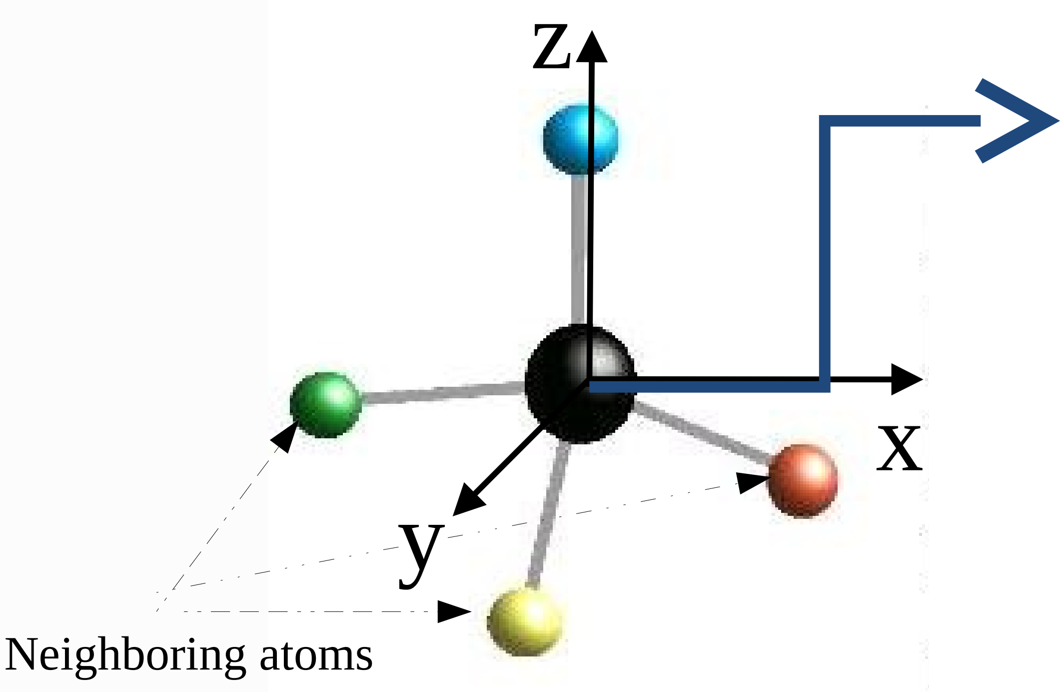
Therefore, a fully quantum and microscopic approach is needed.

III- Quantum approach (nonrelativistic)

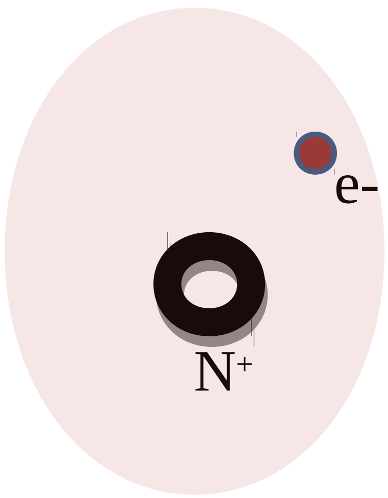
Single-oscillator model

E.U.Condon, Rev.Mod.Phys. (1937)

Optical activity of the molecule is attributed to a single electron in a chiral atom.



chiral atom ≈ Ion (q_N=+e)
+ chromophoric electron (q_e=-e)



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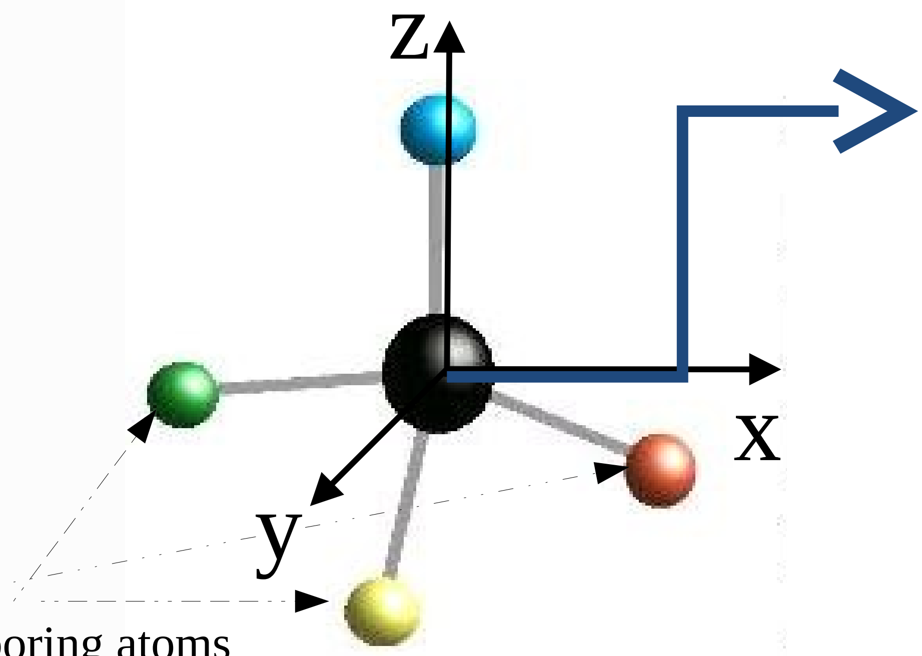

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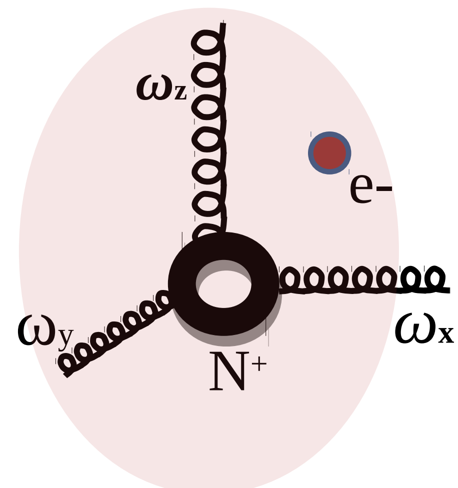

$$V^{HO} = \frac{\mu}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

$\mathbf{r}=\mathbf{r}_N-\mathbf{r}_e$, **p** are the relative position vector and its canonical conjugate momentum.

$\mathbf{R}=(m_N\mathbf{r}_N-m_e\mathbf{r}_e)/M$, **P** are the center of mass position vector and its canonical conjugate momentum

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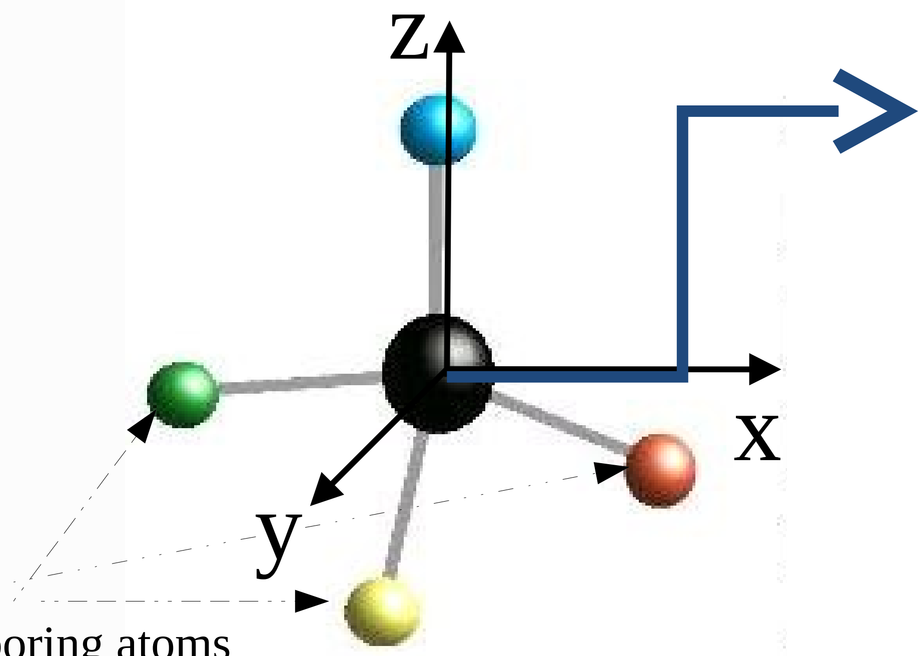

no planes of symmetry
& no inversion centers

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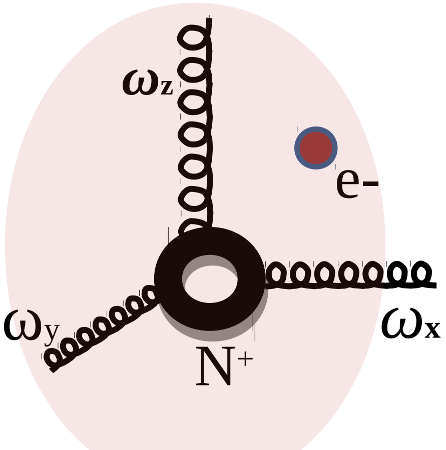

$$V^{HO}=\frac{\mu}{2}(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2)$$

Chiral potential, $V_C = C\,xyz,$ **breaks mirror symmetry.**

III- Quantum approach

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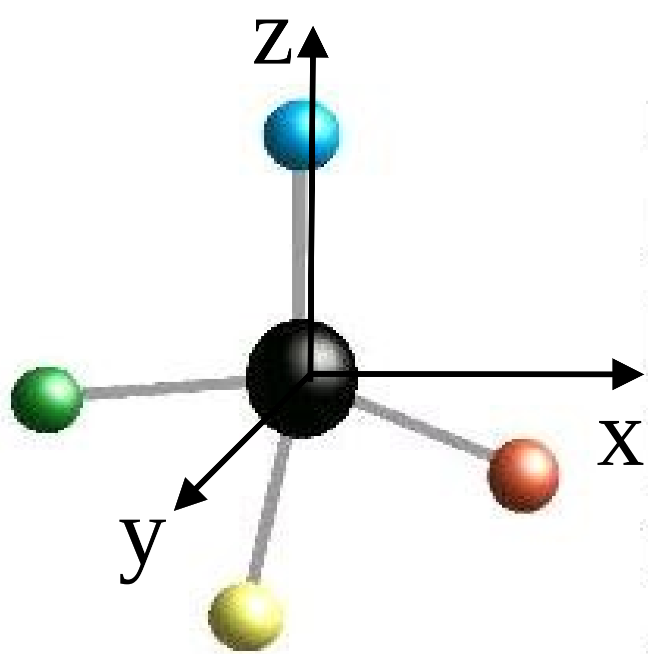
Optical activity of a chiral molecule.

$$\langle \mathbf{d} \rangle = \alpha_E \mathbf{E} - \beta \, \dot{\mathbf{B}}$$

$$\langle \mathbf{m} \rangle = \alpha_M \mathbf{B} + \beta \, \dot{\mathbf{E}}$$



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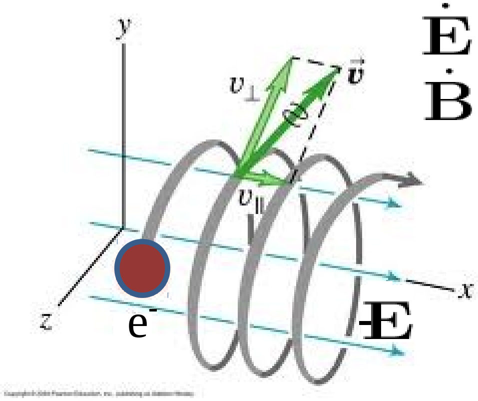

$\dot{\mathbf{E}}$ generates cirular **j** ⇒ **m**
$\dot{\mathbf{B}}$ generates linear -**j** ⇒ **d**

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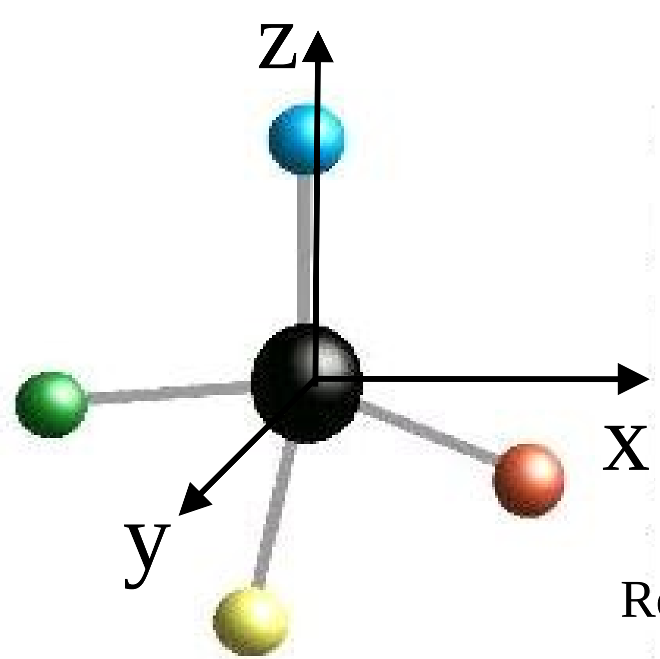

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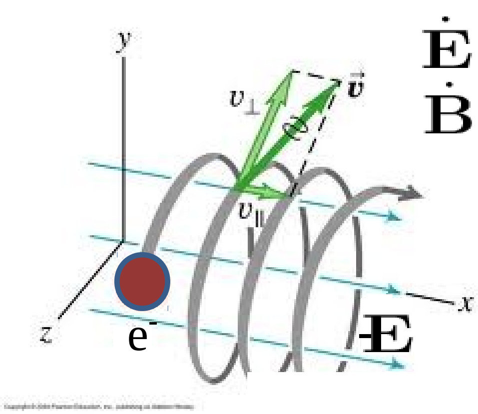
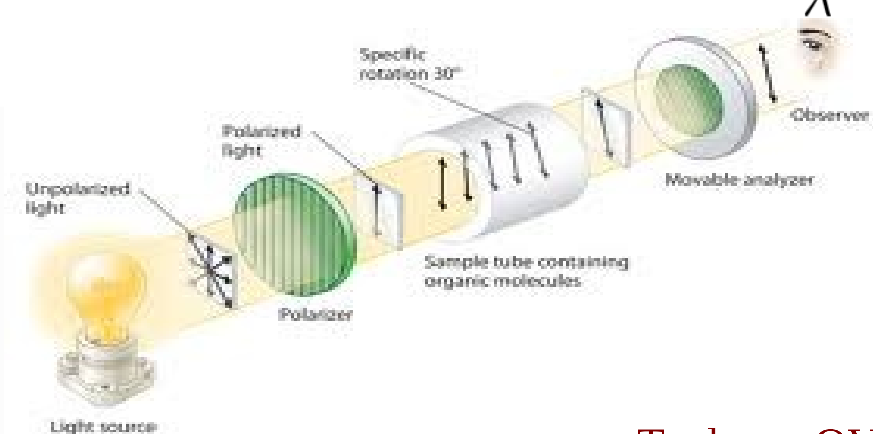


$\dot{\mathbf{E}}$ generates cirular **j** ⇒ **m**
$\dot{\mathbf{B}}$ generates linear -**j** ⇒ **d**

Rotatory power $\varphi = (n_L - n_R)/\lambda = \dfrac{16\pi^2}{\lambda^2}\rho\beta$



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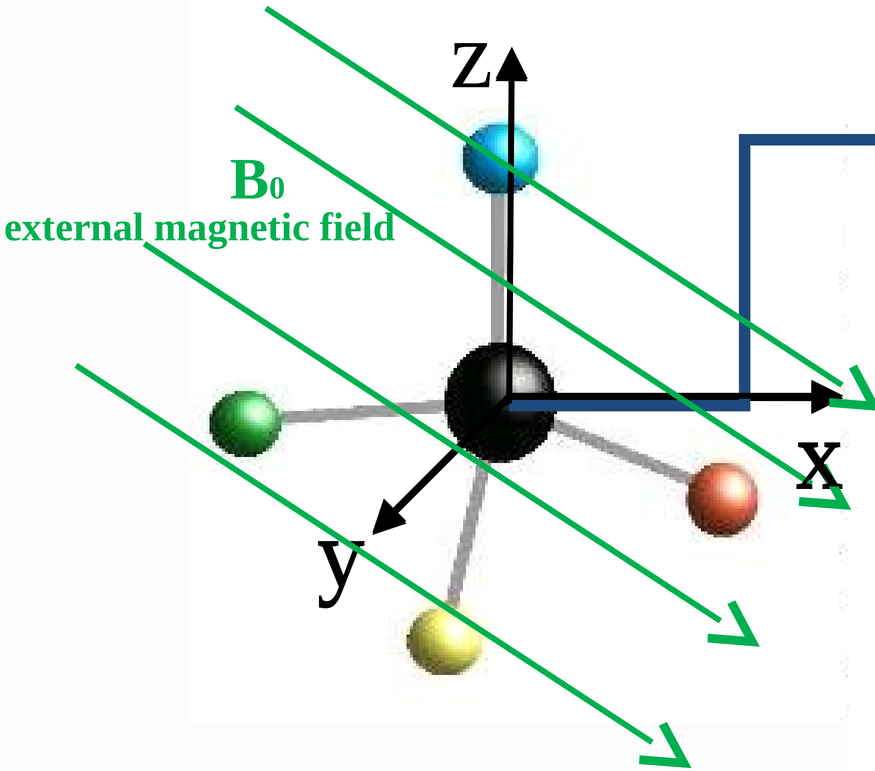

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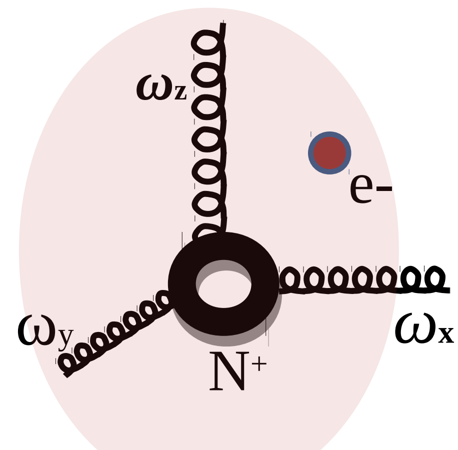

$$V^{HO} = \frac{\mu}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

Zeeman's potential, $V_Z = \frac{e}{2\mu^*}(\mathbf{r} \wedge \mathbf{p}) \cdot \mathbf{B}_0,$

breaks time-reversal symmetry.

III- Quantum approach

Single-oscillator model

.The Total Hamiltonian is $H=H_0+H_F+W$, where H_F is the free field EM Hamiltonian,

$$H_F = \sum_{\mathbf{k},\epsilon} \hbar\omega_{\mathbf{k}}(a^{\dagger}_{\mathbf{k}\epsilon}a_{\mathbf{k}\epsilon} + \frac{1}{2})$$

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.The Hamiltonian of the chiral atom without coupling to the EM quantum field is

$$H_0 = \frac{1}{2\mu}\mathrm{p}^2 + V^{HO} + V_C + V_Z,$$ where

$$V^{HO} = \frac{\mu}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2), \quad V_C = C\,xyz, \quad V_Z = \frac{e}{2\mu^*}(\mathbf{r}\wedge\mathbf{p})\cdot\mathbf{B}_0,$$

III- Quantum approach

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$$H_0 = \frac{1}{2\mu}\mathrm{p}^2 + V^{HO} + V_C + V_Z,$$

.The Hamiltonian of interaction between the chiral atom and the EM vacuum is

$$\begin{aligned} W &= -\frac{e}{m_N}\left(\mathbf{p} + \frac{m_N}{M}\mathbf{P} - \frac{e}{2}\mathbf{B}_0 \wedge \mathbf{r}\right) \cdot \mathbf{A}(\mathbf{R} + \frac{m_e}{M}\mathbf{r}) \\ &- \frac{e}{m_e}\left(\mathbf{p} - \frac{m_e}{M}\mathbf{P} + \frac{e}{2}\mathbf{B}_0 \wedge \mathbf{r}\right) \cdot \mathbf{A}(\mathbf{R} - \frac{m_N}{M}\mathbf{r}) \\ &+ \frac{e^2}{2m_N}\mathrm{A}^2(\mathbf{R} + \frac{m_e}{M}\mathbf{r}) + \frac{e^2}{2m_e}\mathrm{A}^2(\mathbf{R} - \frac{m_N}{M}\mathbf{r}). \end{aligned}$$

III- Quantum approach

Single-oscillator model

.The Total Hamiltonian H possesses a conserved momentum,

$$\boxed{\mathrm{K} = \mathrm{P}_{\mathrm{kin}} + e\mathrm{B}_0 \wedge \mathbf{r} + \mathrm{P}_{\parallel}^{\mathrm{Cas}} + \mathrm{P}_{\perp}^{\mathrm{Cas}}}$$, $[H,\mathbf{K}]=\mathbf{0}$, where

$$\mathbf{P}_{\perp}^{\mathrm{Cas}} = \sum_{\mathbf{k},\epsilon} \hbar\mathbf{k}(a^{\dagger}_{\mathbf{k}\epsilon}a_{\mathbf{k}\epsilon} + 1/2),$$

$$\sim\int \mathrm{d}^3r\, \mathbf{E}_{\perp} \wedge \mathbf{B} \text{ (radiative)}$$

$$\mathbf{P}_{\parallel}^{\mathrm{Cas}} = e[\mathbf{A}_{\perp}(\mathbf{r}_N) - \mathbf{A}_{\perp}(\mathbf{r}_e)].$$

$$\sim\int \mathrm{d}^3r\, \mathbf{E}_{\parallel} \wedge \mathbf{B} \text{ (electrostatic)}$$

III- Quantum approach

Relation between <P^{Cas}> and <P$_{kin}$> in the ground state

$$\langle \mathbf{K} \rangle = \langle \mathbf{P}_{\rm kin} \rangle + e\mathbf{B}_0 \wedge \langle \mathbf{r} \rangle + \langle \mathbf{P}_{\parallel}^{\rm Cas} + \mathbf{P}_{\perp}^{\rm Cas} \rangle \ .$$

III- Quantum approach

Relation between <P^{Cas}> and <P_{kin}> in the ground state

$$\langle\mathbf{K}\rangle = \langle\mathbf{P}_{\rm kin}\rangle + e\mathbf{B}_0\wedge\langle\mathbf{r}\rangle + \langle\mathbf{P}_{\parallel}^{\rm Cas} + \mathbf{P}_{\perp}^{\rm Cas}\rangle\,.$$

.During the switching of $\mathbf{B}_0$, <**K**> remains constant,

$$\frac{\mathrm{d}\langle\mathbf{K}\rangle}{\mathrm{d}t} = i\hbar^{-1}\underset{\overset{\|}{\mathbf{0}}}{\langle[H,\mathbf{K}]\rangle} + e\frac{\partial\mathbf{B}_0(t)}{\partial t}\wedge\underset{\overset{\|}{\mathbf{0}}}{\langle\mathbf{r}\rangle} = \mathbf{0}\,.$$

III- Quantum approach

Relation between <$\mathbf{P}^{\mathrm{Cas}}$> and <$\mathbf{P}_{\mathrm{kin}}$> in the ground state

$$\langle \mathbf{K} \rangle = \langle \mathbf{P}_{\mathrm{kin}} \rangle + e\mathbf{B}_0 \wedge \underset{\substack{\| \\ \mathbf{0}}}{\langle \mathbf{r} \rangle} + \langle \mathbf{P}_{\|}^{\mathrm{Cas}} + \mathbf{P}_{\perp}^{\mathrm{Cas}} \rangle \,.$$

.During the switching of $\mathbf{B}_0$, <**K**> remains constant,

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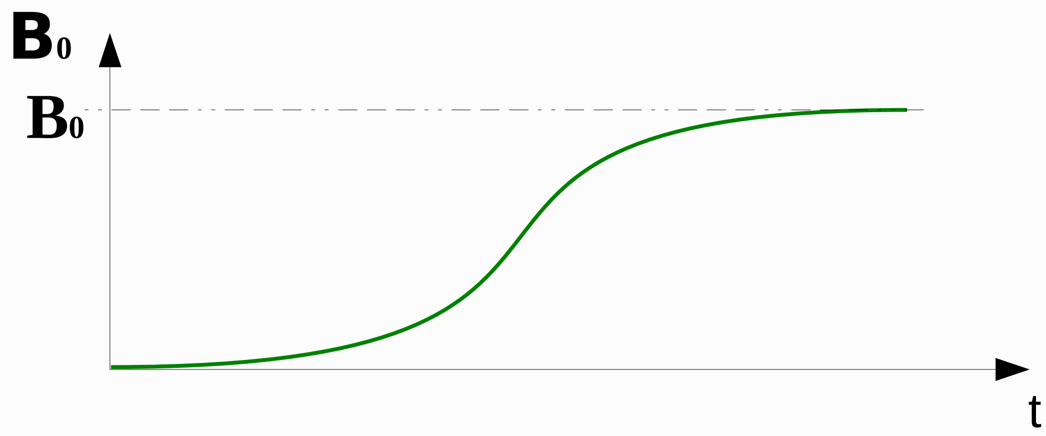
.Therefore, any variation of **<$\mathbf{P}_{\mathrm{kin}}$>** is compensated by a variation of **<$\mathbf{P}^{\mathrm{cas}}$>**,

$$\boxed{\delta \langle \mathbf{P}_{\mathrm{kin}} \rangle = -\delta \langle \mathbf{P}^{\mathrm{Cas}} \rangle.}$$

III- Quantum approach

Relation between <P^{Cas}> and <P_{kin}> in the ground state

.Switching of $\mathbf{B}_0$,



$$\delta\langle\mathbf{P}_{\mathrm{kin}}\rangle = -\delta\langle\mathbf{P}^{\mathrm{Cas}}\rangle \Longrightarrow \Delta\langle\mathbf{P}_{\mathrm{kin}}\rangle = -\int_0^{\mathrm{B}_0}\delta\langle\mathbf{P}^{\mathrm{Cas}}\rangle = -\underbrace{\langle\mathbf{P}^{\mathrm{Cas}}\rangle_{\mathrm{B}_0}}_{\text{in the stationary ground state}}.$$

.3rd order perturbation ($V_C V_Z W$) for $\langle\mathbf{P}_{||}^{\mathrm{Cas}}\rangle$

. 4th order perturbation ($V_C V_Z W^2$) for $\langle\mathbf{P}_{\perp}^{\mathrm{Cas}}\rangle$

III- Quantum approach

Longitudinal momentum, $\mathbf{P}_{||}^{\rm Cas} = e[\mathbf{A}_{\perp}(\mathbf{r}_N) - \mathbf{A}_{\perp}(\mathbf{r}_e)]$,

$$\langle\mathbf{P}_{||}^{\rm Cas}\rangle = \sum_{\mathbf{P},I,\gamma_{\mathbf{k}\epsilon}} \frac{\langle\Omega_0|e[\mathbf{A}(\mathbf{r}_N) - \mathbf{A}(\mathbf{r}_e)]|\mathbf{P},I,\gamma\rangle\langle\mathbf{P},I,\gamma|W|\Omega_0\rangle}{E_0 - E_{P,I,k}} + c.c.$$

ground state of H0

$$-e\mathbf{A}(\mathbf{r_N})\cdot[\mathbf{P}/M + \mathbf{p}/m_N] + e\mathbf{A}(\mathbf{r_e})\cdot[\mathbf{P}/M - \mathbf{p}/m_e]$$

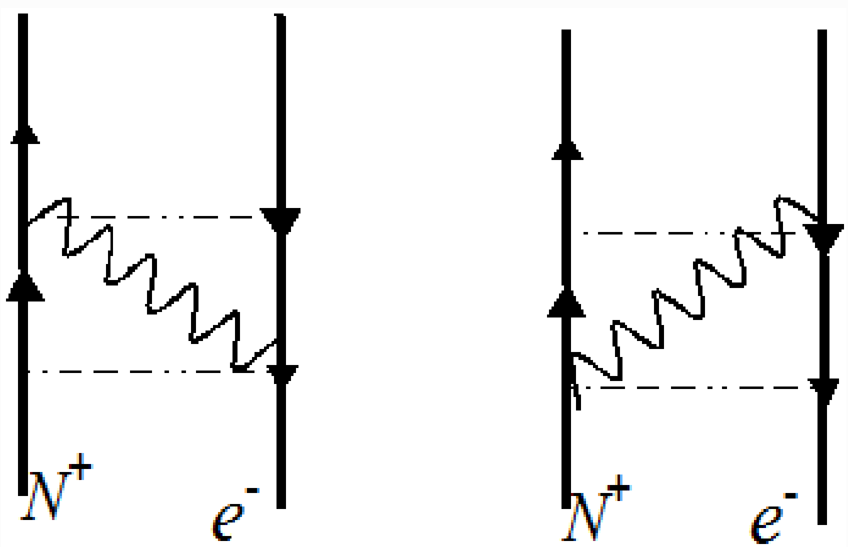
III- Quantum approach

Longitudinal momentum, $\mathbf{P}_{\parallel}^{\text{Cas}} = e[\mathbf{A}_{\perp}(\mathbf{r}_N) - \mathbf{A}_{\perp}(\mathbf{r}_e)]$,

$$\langle \mathbf{P}_{\parallel}^{\text{Cas}} \rangle = \sum_{\mathbf{P},I,\gamma_{\mathbf{k}\epsilon}} \frac{\langle \Omega_0 | e[\mathbf{A}(\mathbf{r}_N) - \mathbf{A}(\mathbf{r}_e)] | \mathbf{P}, I, \gamma \rangle \langle \mathbf{P}, I, \gamma | W | \Omega_0 \rangle}{E_0 - E_{P,I,k}} + c.c.$$

ground state of H_0

$$-e\mathbf{A}(\mathbf{r_N}) \cdot [\mathbf{P}/M + \mathbf{p}/m_N] + e\mathbf{A}(\mathbf{r_e}) \cdot [\mathbf{P}/M - \mathbf{p}/m_e]$$



Subdominant processes. Virtual photons are created and annihilated at different particles.

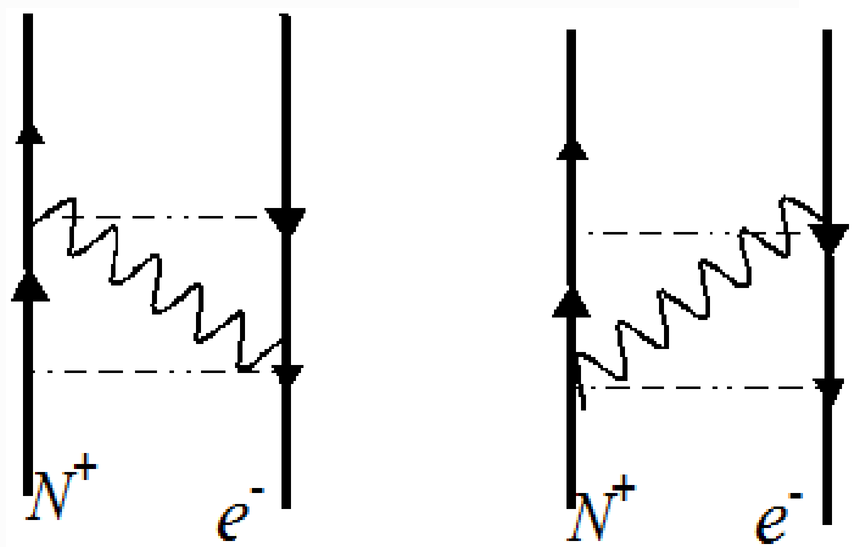
III- Quantum approach

Longitudinal momentum, $\mathbf{P}_{\parallel}^{\mathrm{Cas}} = e[\mathbf{A}_{\perp}(\mathbf{r}_N) - \mathbf{A}_{\perp}(\mathbf{r}_e)]$,

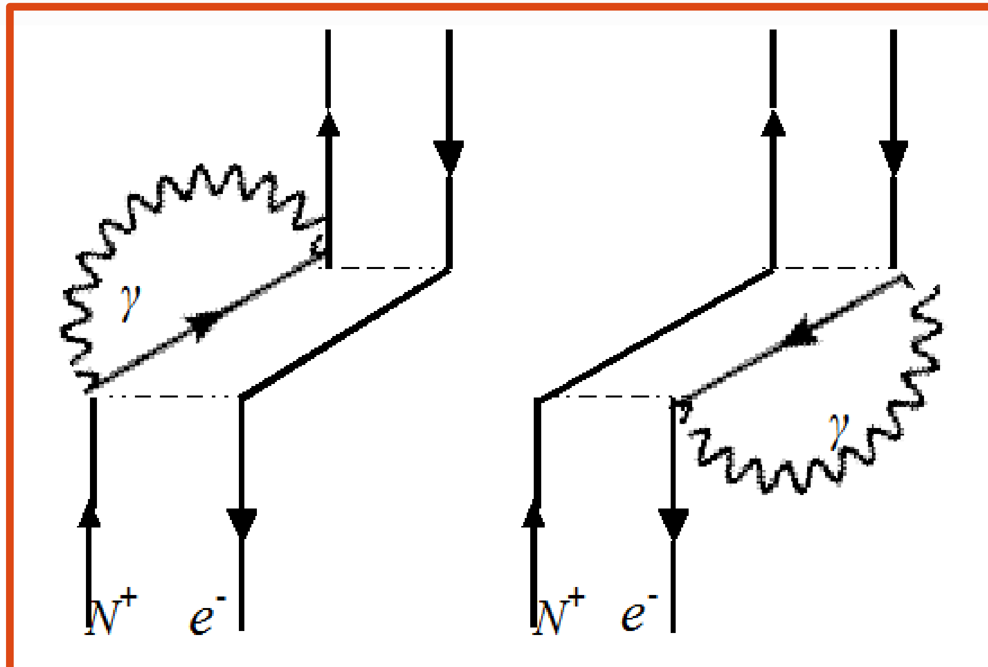
$$\langle \mathbf{P}_{\parallel}^{\mathrm{Cas}} \rangle = \sum_{\mathbf{P}, I, \gamma_{\mathbf{k}\epsilon}} \frac{\langle \Omega_0 | e[\mathbf{A}(\mathbf{r}_N) - \mathbf{A}(\mathbf{r}_e)] | \mathbf{P}, I, \gamma \rangle \langle \mathbf{P}, I, \gamma | W | \Omega_0 \rangle}{E_0 - E_{P,I,k}} + c.c.$$

ground state of H0

$$-e\mathbf{A}(\mathbf{r_N}) \cdot [\mathbf{P}/M + \mathbf{p}/m_N] + e\mathbf{A}(\mathbf{r_e}) \cdot [\mathbf{P}/M - \mathbf{p}/m_e]$$



Subdominant processes. Virtual photons are created and annihilated at different particles.



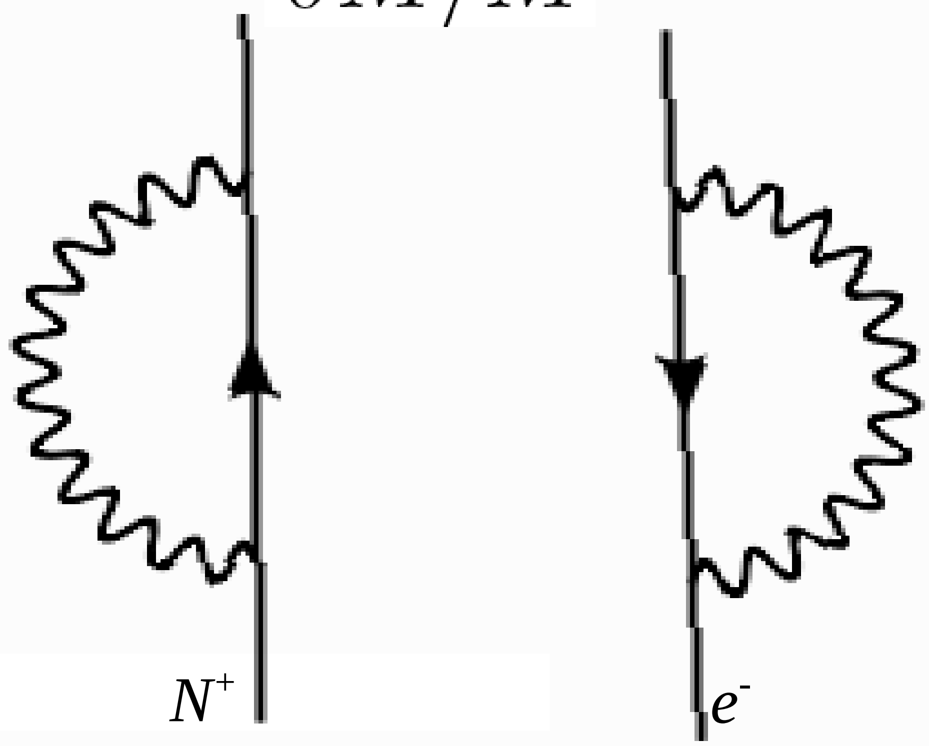
Dominant processes. Virtual photons are created and annihilated at the same particle.

III- Quantum approach

Longitudinal momentum, $\mathbf{P}_{||}^{\rm Cas} = e[\mathbf{A}_{\perp}(\mathbf{r}_N) - \mathbf{A}_{\perp}(\mathbf{r}_e)],$

Mass renormalization terms

$$\underbrace{\frac{4\hbar^2\alpha}{3\pi M}\int k\mathrm{d}k\left[\frac{1}{\hbar^2k^2/2m_e+\hbar ck}+\frac{1}{\hbar^2k^2/2m_N+\hbar ck}\right]}_{\delta M/M}\underbrace{\langle\Omega_0|\mathbf{P}|\Omega_0\rangle}_{<\mathbf{P}_{\rm kin}>}$$

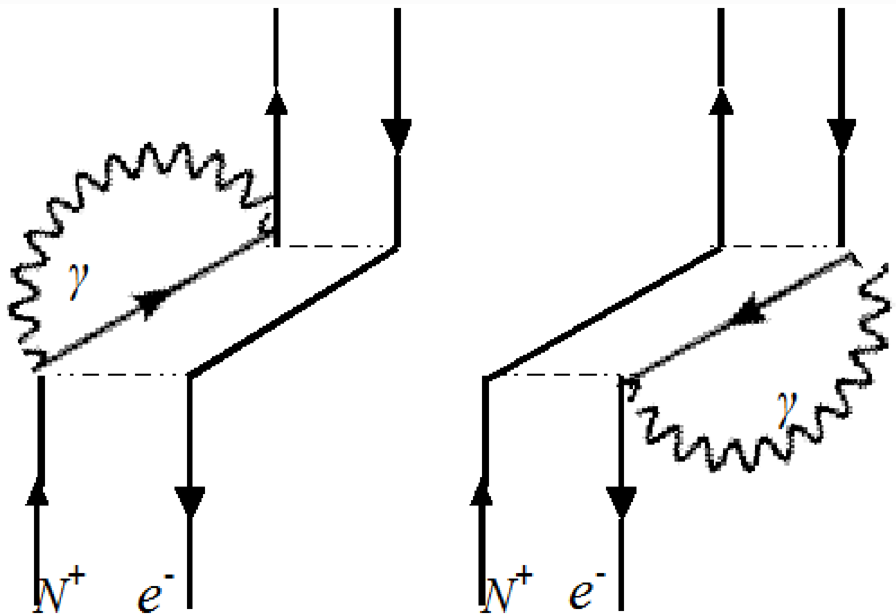


III- Quantum approach

Longitudinal momentum, $\mathbf{P}_{\|}^{\rm Cas}=e[\mathbf{A}_{\perp}(\mathbf{r}_N)-\mathbf{A}_{\perp}(\mathbf{r}_e)],$

$$\langle \mathrm{P}_{\|}^{\rm Cas}\rangle=\frac{\hbar e}{2c\epsilon_0}\int\frac{\mathrm{d}^3k}{(2\pi)^3k}\langle\Omega_0|\frac{(\mathbb{I}-\frac{\mathbf{k}\otimes\mathbf{k}}{k^2})\cdot}{\hbar^2k^2/2m_e+\hbar ck-E_0+H_0}\underbrace{e\mathbf{p}/m_e}_{j(\mathbf{r}_e)\,\|\,\mathbf{B}_0}|\Omega_0\rangle+c.c.-[m_e\rightarrow m_N]$$

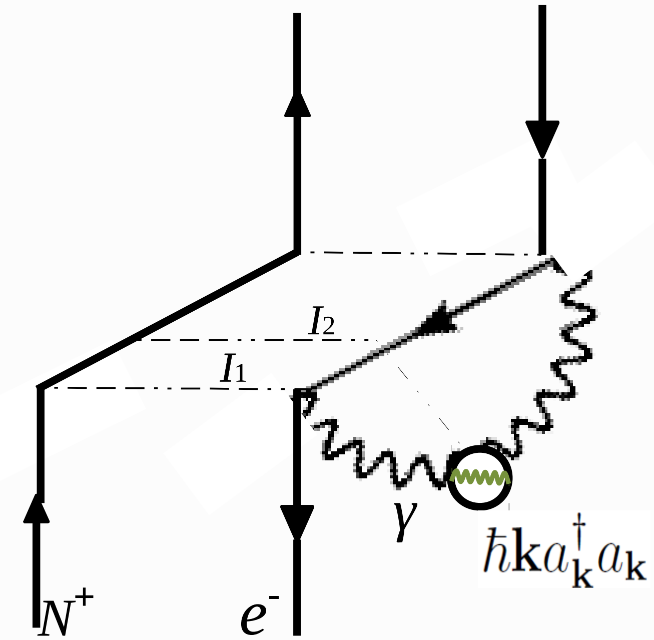
$$=\int\mathrm{d}^3x\,\langle\mathbf{E}_{\|}(\mathbf{x}-\mathbf{r}_e)\wedge\mathbf{B}(\mathbf{x}-\mathbf{r}_e)\rangle+\langle\mathbf{E}_{\|}(\mathbf{x}-\mathbf{r}_N)\wedge\mathbf{B}(\mathbf{x}-\mathbf{r}_N)\rangle$$



III- Quantum approach

Transverse momentum, $\mathbf{P}_{\perp}^{\mathrm{Cas}} = \sum_{\mathbf{k},\epsilon} \hbar\mathbf{k}(a^{\dagger}_{\mathbf{k}\epsilon}a_{\mathbf{k}\epsilon} + 1/2)$

$$\langle \mathbf{P}_{\perp}^{\mathrm{Cas}} \rangle = \frac{\hbar^2 e^2}{2c\epsilon_0 m_e^2} \int \frac{\mathrm{d}^3 k \, \mathbf{k}}{(2\pi)^3 k} \langle \Omega_0 | (\mathbf{p} + \frac{e}{2}\mathbf{B}_0 \wedge \mathbf{r}) e^{-i\frac{m_N}{M}\mathbf{k}\cdot\mathbf{r}} \frac{\cdot(\mathbb{I} - \frac{\mathbf{k}\otimes\mathbf{k}}{k^2})\cdot}{(\hbar^2 k^2/2M + \hbar c k - E_0 + H_0)^2}$$
$$\times \quad e^{i\frac{m_N}{M}\mathbf{k}\cdot\mathbf{r}} (\mathbf{p} + \frac{e}{2}\mathbf{B}_0 \wedge \mathbf{r}) | \Omega_0 \rangle$$



III- Quantum approach

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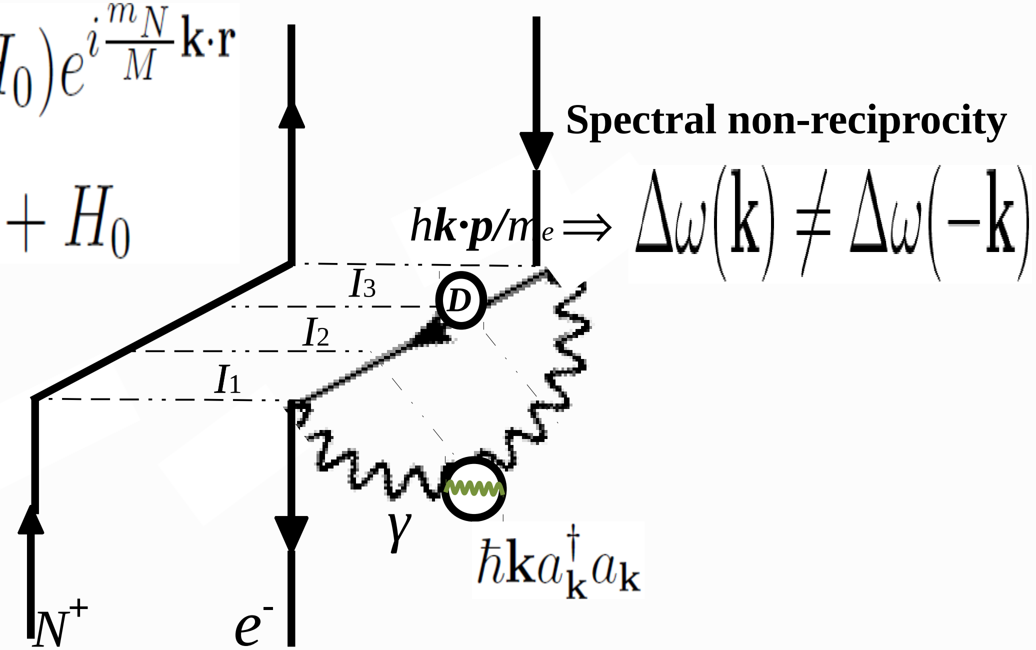
$$\langle \mathbf{P}_{\perp}^{\text{Cas}} \rangle = \frac{\hbar^2 e^2}{2c\epsilon_0 m_e^2}\int \frac{\mathrm{d}^3k\,\mathbf{k}}{(2\pi)^3 k}\langle\Omega_0|(\mathbf{p}+\frac{e}{2}\mathbf{B}_0\wedge\mathbf{r})e^{-i\frac{m_N}{M}\mathbf{k}\cdot\mathbf{r}}\frac{\cdot(\mathbb{I}-\frac{\mathbf{k}\otimes\mathbf{k}}{k^2})\cdot}{(\hbar^2k^2/2M+\hbar ck-E_0+H_0)^2}$$
$$\times\; e^{i\frac{m_N}{M}\mathbf{k}\cdot\mathbf{r}}(\mathbf{p}+\frac{e}{2}\mathbf{B}_0\wedge\mathbf{r})|\Omega_0\rangle$$

.Doppler shift caused by electron's motion yields *spectral non-reciprocity*.

$$e^{-i\frac{m_N}{M}\mathbf{k}\cdot\mathbf{r}}(\hbar^2k^2/2M+\hbar ck-E_0+H_0)e^{i\frac{m_N}{M}\mathbf{k}\cdot\mathbf{r}}$$
$$= \hbar^2k^2/2m_e+\hbar ck\boxed{-\hbar\mathbf{k}\cdot\mathbf{p}/m_e}-E_0+H_0$$



III- Quantum approach

Quantum computation of δ<P^{cas}>

$$\langle \mathbf{P}_{\perp}^{\mathrm{Cas}} \rangle = \frac{-Ce^3 \mathbf{B}_0}{144\pi^2 c\epsilon_0 m_e^2 \omega_x \omega_y \omega_z} \eta^{zy}\eta^{xz}\eta^{yx}$$

$$\langle \mathbf{P}_{\parallel}^{\mathrm{Cas}} \rangle = \frac{Ce^3 \ln{(m_e/m_N)} \mathbf{B}_0}{144\pi^2 c\epsilon_0 \mu^2 \omega_x \omega_y \omega_z} \eta^{zy}\eta^{xz}\eta^{yx}$$

$$\delta \langle \mathrm{P}_{\parallel}^{\mathrm{Cas}} \rangle \simeq 10 \delta \langle \mathrm{P}_{\perp}^{\mathrm{Cas}} \rangle$$

$$\eta^{ij} = \frac{\omega_i - \omega_j}{\omega_i + \omega_j}$$

III- Quantum approach

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$$\sim e\alpha\frac{\beta(0)}{\alpha_E(0)}$$

fine structure const.

static rotatory power

static electrical polarizability

arXiv:1307.6611 M.D., G.Rikken&B.van-Tiggelen

III- Quantum approach

Result and experimental test

$$\boxed{\langle \mathbf{P}^{\rm Cas}\rangle \simeq \frac{2\alpha}{9\pi}\frac{\beta(0)}{\alpha_E(0)}[\ln{(m_N/m_e)}+1]e\mathbf{B}_0}$$

Eg., 2-octanol, $\mathrm{C_8H_{18}O}$, from tabulated data on refractive index and rotatory power, for $\mathbf{B}_0$=10T, $\Delta\mathbf{v}\sim$ 1nm/s.

For the same model, the semi-classical approach using the FDT

$\langle \mathbf{P}^{\rm Cas}\rangle = \frac{\hat{\mathbf{k}}\hbar}{3\pi^2c^4}\int_0^\infty \gamma_\perp\omega^3\mathrm{d}\omega$ yields 9 orders of magnitude less!

III- Quantum approach

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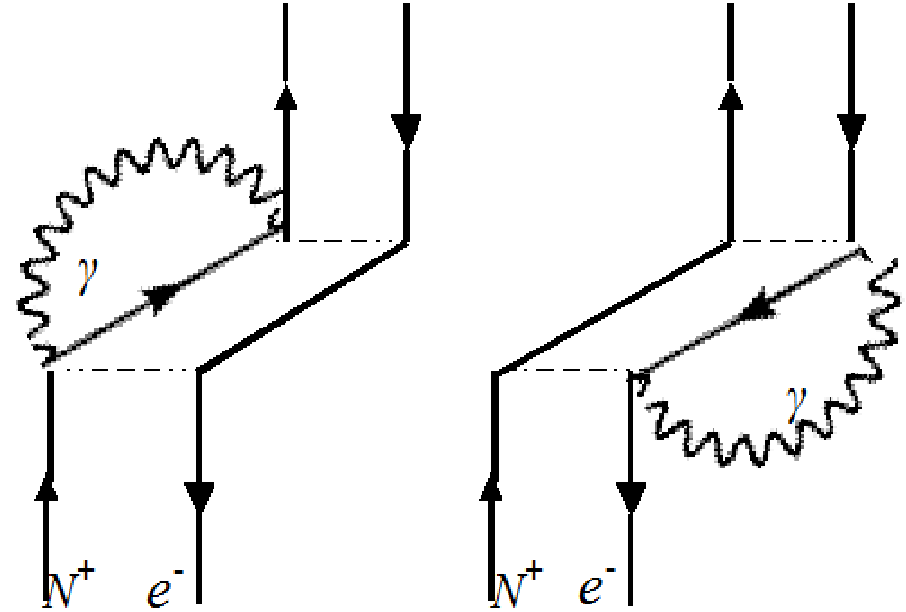
Where does the transferred kinetic energy come from?

III- Quantum approach

Interaction energy

i) A first interaction energy includes crossed terms **pA** – **PA**, with photons created and annihilated at the same particle,

$$\langle W_{\|}\rangle = \sum_{\mathbf{P},I,\gamma_{\mathbf{k}\epsilon}} \frac{\langle\Omega_0|-e\mathbf{p}\cdot\mathbf{A}(\mathbf{r}_e)/m_e|\mathbf{P},I,\gamma\rangle\langle\mathbf{P},I,\gamma|e\mathbf{P}\cdot\mathbf{A}(\mathbf{r}_e)/M|\Omega_0\rangle}{E_0-E_{P,I,k}} + c.c. - [m_e \rightarrow m_N]$$



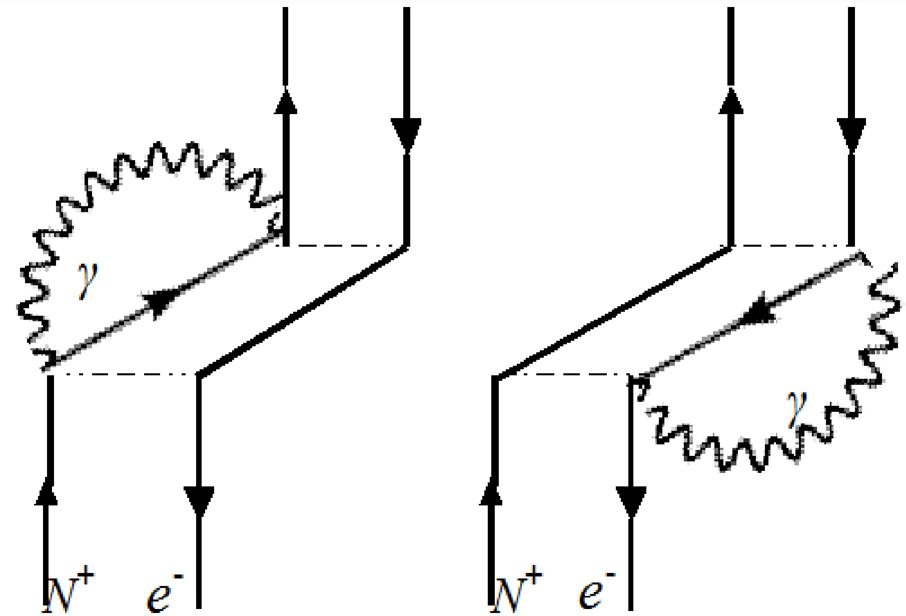
III- Quantum approach

Interaction energy

i) A first interaction energy includes crossed terms **pA** – **PA**, with photons created and annihilated at the same particle,

$$\langle W_{\parallel}\rangle = \sum_{\mathbf{P},I,\gamma_{\mathbf{k}\epsilon}} \frac{\langle\Omega_0| - e\mathbf{p}\cdot\mathbf{A}(\mathbf{r}_e)/m_e|\mathbf{P},I,\gamma\rangle\langle\mathbf{P},I,\gamma|e\mathbf{P}\cdot\mathbf{A}(\mathbf{r}_e)/M|\Omega_0\rangle}{E_0 - E_{P,I,k}} + c.c. - [m_e \rightarrow m_N]$$

$$= \underbrace{-\langle\mathbf{P}_{\parallel}^{\mathrm{Cas}}\rangle}\cdot\langle\mathbf{P}_{\mathrm{kin}}\rangle/M$$



III- Quantum approach

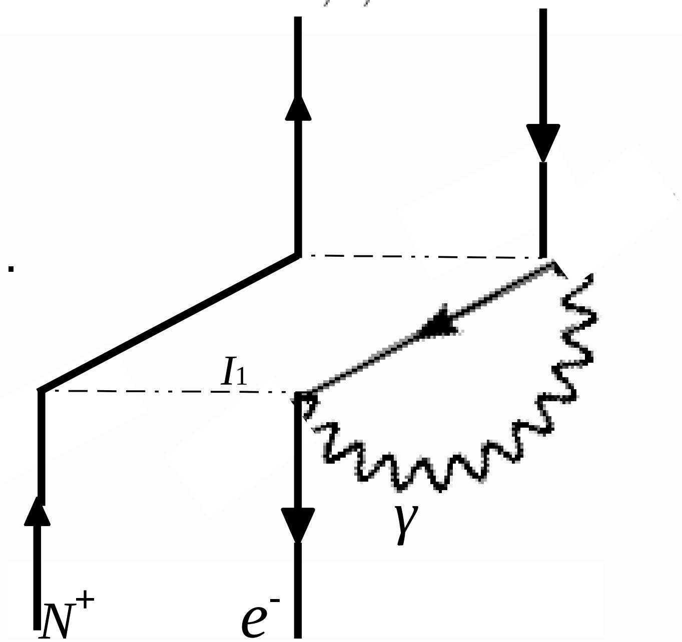
Interaction energy

ii) A second interaction energy comes from the Doppler shift correction term $h\mathbf{k}\mathbf{P}_{\mathrm{kin}}/M$ to the usual Lamb shift **~** <**pA** – **pA**>,

$$\langle W_\perp\rangle = \sum_{\mathbf{P},I,\gamma_{\mathbf{k}\epsilon}} \frac{\langle\Omega_0|e\mathbf{p}\cdot\mathbf{A}(\mathbf{r}_e)/m_e|\mathbf{P},I,\gamma\rangle\langle\mathbf{P},I,\gamma|e\mathbf{p}\cdot\mathbf{A}(\mathbf{r}_e)/M|\Omega_0\rangle}{E_0 - E^{\text{CM-Doppler}}_{P,I,k}}$$

$$= \Delta E^{\text{CM-Doppler}}_{\text{Lamb}}$$

$$= -\langle\mathbf{P}^{\mathrm{Cas}}_\perp\rangle\cdot\langle\mathbf{P}_{\mathrm{kin}}\rangle/M.$$



III- Quantum approach

Interaction energy

The work done over the system during the adiabatic switching of the magnetic field equals the kinetic energy of the center of mass,

$$\int_0^{\mathbf{B}_0} \delta\langle W_{||}\rangle + \delta\langle W_{\perp}\rangle = \mathbf{P}^2_{\rm kin}(\mathbf{B}_0)/2M.$$

IV- Conclusions

. We have shown that the simultaneous breakdown of T&P symmetries allows for the transfer of net momentum from the quantum vacuum to matter.

It is a one-loop quantum effect, like the Casimir effect or the Lamb-shift.

. A chiral molecule in a magnetic field acquires a kinetic momentum directed along $\mathbf{B}_0$ and proportional to the fine structure const. and the rotatory power,

$$\langle \mathbf{P}^{\mathrm{Cas}} \rangle \simeq \frac{2\alpha}{9\pi} \frac{\beta(0)}{\alpha_E(0)} [\ln{(m_N/m_e)} + 1] e\mathbf{B}_0 \; .$$

. We conjecture this result is universal --up to factors of order unity.

. For common chiral compounds we estimate $\Delta\mathbf{v} \sim 1$nm/s for $\mathbf{B}_0$=10T.

. A semi-classical approach based on the FDT fails.

IV- Conclusions

.Two mechanisms operate in this effect:

i) In $\mathbf{P}_{\parallel}^{\mathrm{Cas}}$, it is the coupling of the longitudinal field of the charges to the magnetic field generated by their unbalanced currents, $\int \mathrm{d}^3x\,\langle \mathbf{E}_{\parallel}(\mathbf{x}-\mathbf{r}_q)\wedge \mathbf{B}(\mathbf{x}-\mathbf{r}_q)\rangle$.

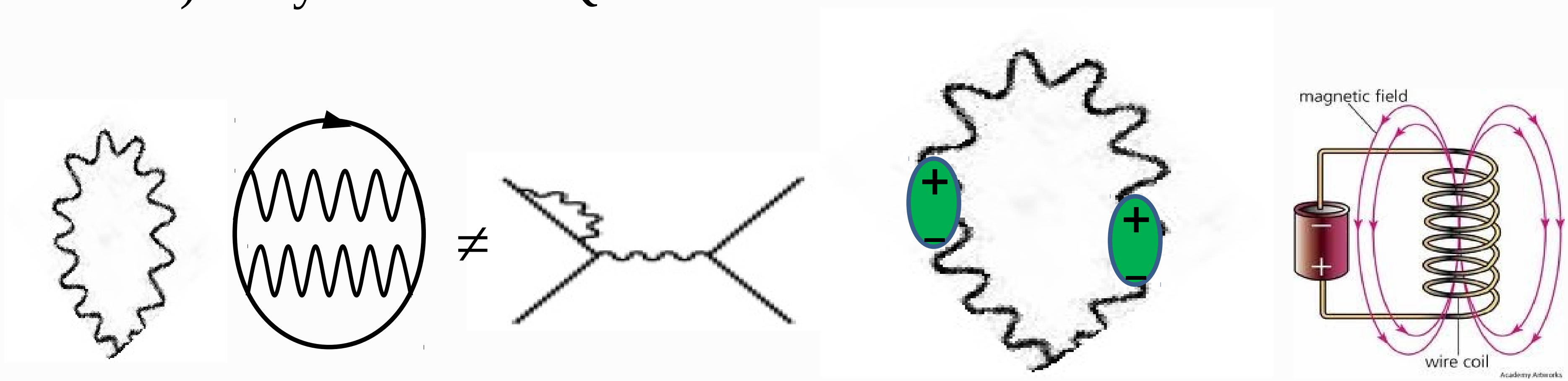
ii) In $\mathbf{P}_{\perp}^{\mathrm{Cas}}$, it is the Doppler shift due to the internal motion of charges that yields the spectral non-reciprocity for transverse modes, $\Delta\omega(\mathbf{k}) \neq \Delta\omega(-\mathbf{k})$.

. Kinetic energy is supplied by external magnetic field.

IV- Conclusions

. Prospective work on

i) Fully relativistic QED calculation.



ii) Vacuum in the lab: Chiral superconductors.
Observable: Bias supercurrent. Energy from condensation.

iii) Vacuum in QFT-HEP: Electroweak theory.
T,P odd observable? Coupling to EM?