Perturbed de Sitter

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The question : **Lambda or not Lambda ?**

- gravitation (& Lambda) vs cosmology (observations)

- general considerations on the « role » of Lambda

- local effects of Lambda ?

  → a model that shows it could be responsible of anisotropies

I – Cosmological context

Context : Accelerated expansion of the universe interpreted in the

General Relativity with cosmological constant (LGR) framework

→ Concordance LambdaCDM (LCDM) model

LCDM **advantages** … :

- well known & tested physics : gravitation / general relativity
  (but with a cosmological constant ←→ vacuum as perfect fluid $P = -\varepsilon$)

- **the model works very well !** (SN1a, CMBR, …)

- refers (may refer …) to vacuum energy in physics (Casimir effect, ….)

… but **unsolved problems** :

- bad interface with quantum field theory : **120 (60 ?) orders between the cosmological Lambda & its QFT expected value** (vacuum energy)
- coïncidence pb, …

→ some authors prefer other solutions (suggested by LGR equation)

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta} \quad \leftrightarrow \quad R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = 8\pi T_{\alpha\beta} + 8\pi \widetilde{T}_{\alpha\beta} \left(\widetilde{P} = -\widetilde{\varepsilon}\right)$$

change **gravity** theory	- ~~general relativity~~ → gravity = scalar-tensor, *f*(*R*), ….
change **matter** content	- matter content includes exotic (dark) matter/energy
change nothing, but remove **symetries**	- inhomogeneities (voids, ….)
	- ….

II – General considerations on the cosmological constant effects

Discarding here this controversy, the fact the interpretation in terms of Λ results in a valuable cosmological scenario raises the question :

could Λ result in observable **effects at scales smaller than cosmological scales** ?

no Λ clustering effect ➔ cosmo amplitude → amplitude for all scales (in some sense…)

Works made along these lines (LGR) :

matter

- motions about black holes → incidences on accretion disks (?) [refs …]
- gravitational equilibrium [refs …]
- solar system : periastron shift, … [refs …]
- weak local value of the Hubble parameter (~ 60 km/s/Mpc vs ~ 70) [refs …]

light

- lensing [refs …]

Often expected local effect : the common wisdom says

« the cosmological constant acts as a **radial** repulsive force proportional to the distance »

A general proof of this claim ??????

Supported by **Schwarzschild-de Sitter** solution …

$$ds^2 = -\left(1-\underbrace{\frac{2m}{r}-\frac{\Lambda r^2}{3}}_{\text{weak field}}\right)dt^2 + \left(1-\frac{2m}{r}-\frac{\Lambda r^2}{3}\right)^{-1}dr^2 + r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)$$

$$\vec{g}_{eff} = -m\frac{\vec{r}}{r^3} + \boxed{\frac{\Lambda}{3}\vec{r}}$$

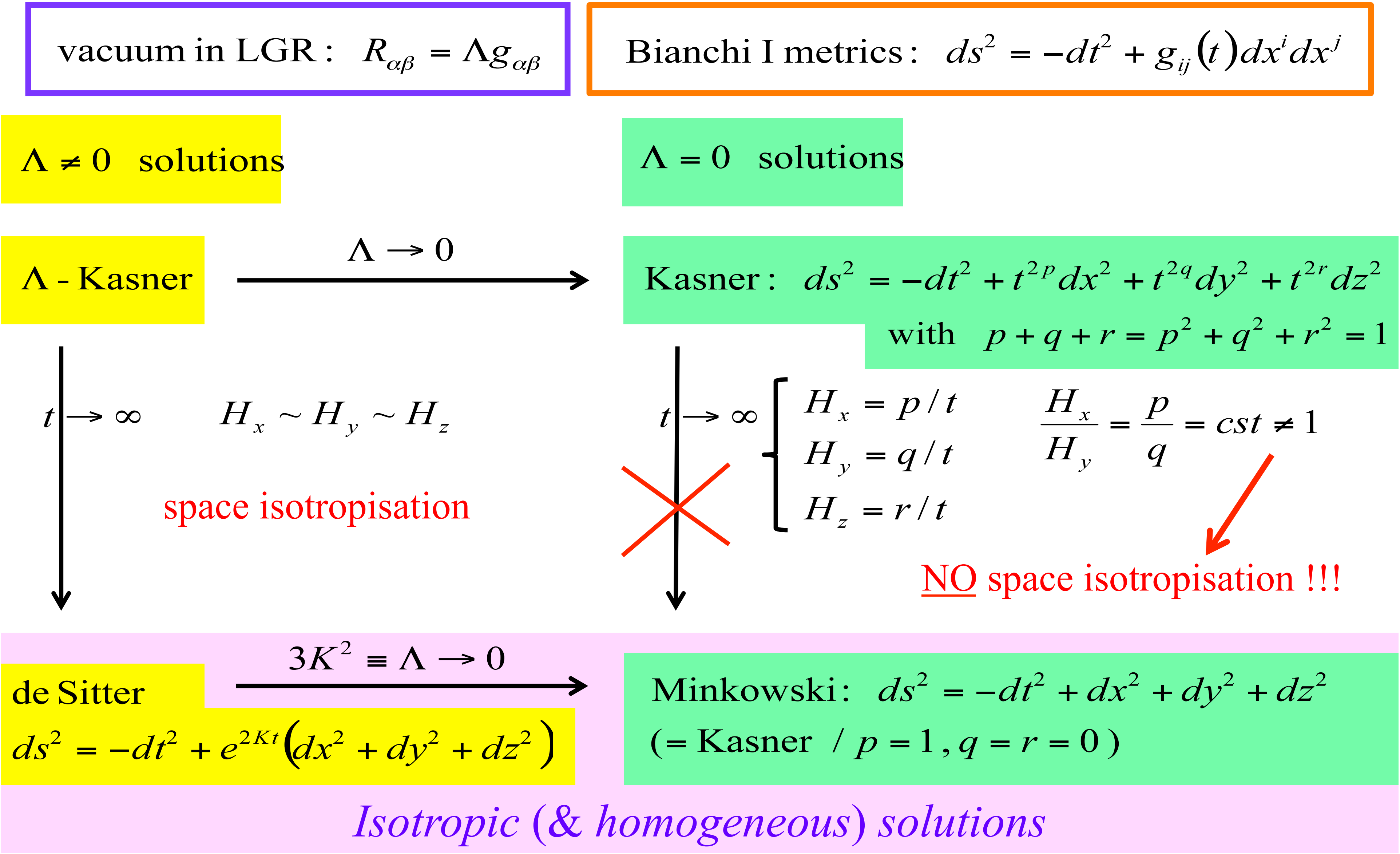
… and by **RW-cosmological models** (including the Einstein static universe) ...

… but in all these models, the **spherical symmetry is present from the very start !!!**

… and … **solutions are known that do not share this property** (Lambda-Kassner)

➔ Λ **may result in non-spherical effects**

Λ vs **vacuum**
homogeneous solutions



vacuum in LGR : $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$

Bianchi I metrics : $ds^2 = -dt^2 + g_{ij}(t)dx^i dx^j$

$\Lambda \neq 0$ solutions

$\Lambda = 0$ solutions

Λ - Kasner

$\Lambda \to 0$

Kasner : $ds^2 = -dt^2 + t^{2p}dx^2 + t^{2q}dy^2 + t^{2r}dz^2$

with $p + q + r = p^2 + q^2 + r^2 = 1$

$t \to \infty$

$H_x \sim H_y \sim H_z$

space isotropisation

$t \to \infty$

$$\begin{cases} H_x = p/t \\ H_y = q/t \\ H_z = r/t \end{cases} \qquad \frac{H_x}{H_y} = \frac{p}{q} = cst \neq 1$$

<u>NO</u> space isotropisation !!!

$3K^2 \equiv \Lambda \to 0$

de Sitter

$ds^2 = -dt^2 + e^{2Kt}\left(dx^2 + dy^2 + dz^2\right)$

Minkowski : $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$

(= Kasner / $p = 1, q = r = 0$)

Isotropic (& homogeneous) solutions

→ At the cosmological level, a non-zero cosmological constant drives a vacuum (asympt vac ?) expanding (Bianchi I) universe into an isotropic state

III - How to determine the general Lambda effect ?

Preliminary study : expand LGR equation $R_{\alpha\beta} = 8\pi\left(T_{\alpha\beta} - \frac{1}{2}Tg_{\alpha\beta}\right) + \Lambda g_{\alpha\beta}$

- about Minkowski $g_{\alpha\beta} = m_{\alpha\beta} + h_{\alpha\beta}$ with $\left|h_{\alpha\beta}\right| << 1$
- without any prior symmetry assumption

→ OK for **local effects** (← Minkowski is NOT a LGR (vacuum) solution) …

Not necessarily isotropic …. (Chauvineau & Regimbau, 2012)

But : what if **more than** just **local** questions are into consideration ?

For instance, if one has to :

- match local (anisotropic) effects to (isotropic) cosmological expansion ?
- consider in a coherent way local effects here & there ?

If **more than** just **local** ➔ **expansion about an exact LGR** is required

➔ choice : expand about **de Sitter** :

- **simplest** exact LGR
- cosmological context
- vacuum ($T_{\alpha\beta} = 0$) → isolate **vacuum** (w.r.t. matter) **effects**

→ **impact** of perturbations **on <u>*z* distribution</u>** (cosmological observable)

De Sitter in Robertson-Walker coordinates

$$\boxed{ds^2 = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right) \ \text{with} \ \ a = e^{Kt}} \quad \& \quad K = \sqrt{\frac{\Lambda}{3}}$$

$$\tilde{t}(t,r) = \ldots$$

$$\tilde{r}(t,r) = \ldots$$

$$ds^2 = -\left(1 - \frac{\Lambda\tilde{r}^2}{3}\right)d\tilde{t}^2 + \left(1 - \frac{\Lambda\tilde{r}^2}{3}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

→ perturbed metric (use gauge freedom to maintain synchronous coord.)

$$\boxed{ds^2 = -dt^2 + a^2\left[\delta_{ij} + \theta_{ij}\left(t, x^k\right)\right]dx^i dx^j \quad \text{with} \quad \left|\theta_{ij}\right| << 1} \qquad \left(x^1, x^2, x^3\right) \equiv \left(x, y, z\right)$$

→ Insert in $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$ & linearize → a way to (linearized) solutions :

Choose any $V\left(x,y,z\right)$ & $\Psi_i\left(x,y,z\right) \xrightarrow{U\text{ def}} 4K^2U\left(x,y,z\right) \equiv div\left(\vec{\Psi} - \vec{\partial}V\right)$

$$\theta_{kk} = a^{-2}U + V$$

$$\partial_k\theta_{ik} = a^{-2}\partial_i U + \Psi_i$$

$$\partial_k\partial_k\theta_{ij} - a^{-1}\partial_t\left(a^3\partial_t\theta_{ij}\right) = a^{-2}\partial_i\partial_j U + \partial_i\Psi_j + \partial_j\Psi_i - 2K^2U\delta_{ij} - \partial_i\partial_j V$$

Linear system → general solution = particular sol. + homogeneous general sol.

ok (in spatial-Fourier form)

$$\left.\begin{aligned}\theta_{ij}(t,\vec{x}) = &\int\left[\widetilde{C}_{ij}\left(\frac{\mu}{a}\sin\frac{\mu}{a}+\cos\frac{\mu}{a}\right)+\overline{C}_{ij}\left(\frac{\mu}{a}\cos\frac{\mu}{a}-\sin\frac{\mu}{a}\right)\right]\cos\left(K\vec{\mu}\vec{x}\right)d^3\vec{\mu}\\ &+\int\left[\widetilde{S}_{ij}\left(\frac{\mu}{a}\sin\frac{\mu}{a}+\cos\frac{\mu}{a}\right)+\overline{S}_{ij}\left(\frac{\mu}{a}\cos\frac{\mu}{a}-\sin\frac{\mu}{a}\right)\right]\sin\left(K\vec{\mu}\vec{x}\right)d^3\vec{\mu}\end{aligned}\right\}\ \boxed{\text{homog sol}}$$

$$\left.\begin{aligned}&+\frac{1}{a^2}\int\left[\widetilde{P}_{ij}\cos\left(K\vec{\mu}\vec{x}\right)+\overline{P}_{ij}\sin\left(K\vec{\mu}\vec{x}\right)\right]d^3\vec{\mu}\\ &+\int\left[\left(\frac{\widetilde{Q}_{ij}}{K^2}+2\widetilde{P}_{ij}\right)\cos\left(K\vec{\mu}\vec{x}\right)+\left(\frac{\overline{Q}_{ij}}{K^2}+2\overline{P}_{ij}\right)\sin\left(K\vec{\mu}\vec{x}\right)\right]\frac{d^3\vec{\mu}}{\mu^2}\end{aligned}\right\}\ \boxed{\text{particular sol}}$$

where

$\widetilde{C}_{ij},\overline{C}_{ij},\widetilde{S}_{ij},\overline{S}_{ij}\left(\vec{\mu}\right)$
having the form

$$\begin{pmatrix}B_{11}\\B_{22}\\B_{33}\\B_{12}\\B_{23}\\B_{13}\end{pmatrix}=B\,.\begin{pmatrix}2\mu_1\mu_2\mu_3\cos\Phi\\ 2\mu_1\mu_2\mu_3\cos\left(\Phi+2\pi/3\right)\\ 2\mu_1\mu_2\mu_3\cos\left(\Phi+4\pi/3\right)\\ \mu_3\mu_3\mu_3\cos\left(\Phi+4\pi/3\right)-\mu_1\mu_1\mu_3\cos\Phi-\mu_2\mu_2\mu_3\cos\left(\Phi+2\pi/3\right)\\ \mu_1\mu_1\mu_1\cos\Phi-\mu_1\mu_2\mu_2\cos\left(\Phi+2\pi/3\right)-\mu_1\mu_3\mu_3\cos\left(\Phi+4\pi/3\right)\\ \mu_2\mu_2\mu_2\cos\left(\Phi+2\pi/3\right)-\mu_2\mu_3\mu_3\cos\left(\Phi+4\pi/3\right)-\mu_1\mu_1\mu_2\cos\Phi\end{pmatrix}$$

$$\widetilde{P}_{ij}=\frac{1}{4}\mu_i\mu_j\left(\widetilde{V}+\frac{\mu_k\overline{\Psi}_k}{K\mu^2}\right)$$

$$\overline{P}_{ij}=\frac{1}{4}\mu_i\mu_j\left(\overline{V}-\frac{\mu_k\widetilde{\Psi}_k}{K\mu^2}\right)$$

$$\widetilde{Q}_{ij}=K^2\widetilde{V}\left(\frac{1}{2}\delta_{ij}\mu^2-\mu_i\mu_j\right)-K\left(\mu_i\overline{\Psi}_j+\mu_j\overline{\Psi}_i-\frac{1}{2}\delta_{ij}\mu_k\overline{\Psi}_k\right)$$

$$\overline{Q}_{ij}=K^2\overline{V}\left(\frac{1}{2}\delta_{ij}\mu^2-\mu_i\mu_j\right)+K\left(\mu_i\widetilde{\Psi}_j+\mu_j\widetilde{\Psi}_i-\frac{1}{2}\delta_{ij}\mu_k\widetilde{\Psi}_k\right)$$

for any

$B=\widetilde{C},\overline{C},\widetilde{S},\overline{S}\left(\vec{\mu}\right)$

$\Phi=\widetilde{\Phi}_C,\overline{\Phi}_C,\widetilde{\Phi}_S,\overline{\Phi}_S\left(\vec{\mu}\right)$

$\widetilde{V},\overline{V},\widetilde{\Psi}_i,\overline{\Psi}_i\left(\vec{\mu}\right)$

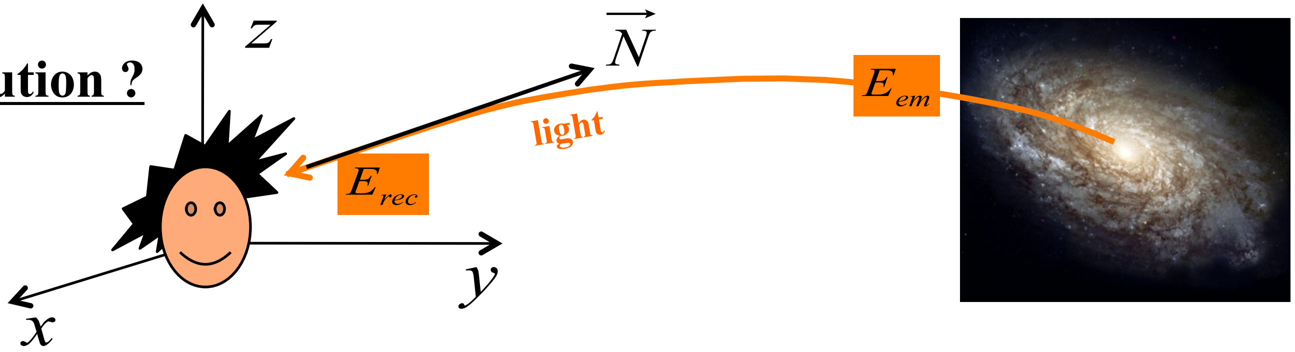
$$ds^2 = -dt^2 + a^2\left(\delta_{ij} + \theta_{ij}\right)dx^i dx^j \quad \text{with} \quad \left|\theta_{ij}\right| << 1$$

isotropic part (de Sitter)

anisotropic part → Expected anisotropic incidence on redshift distribution

Induced redshift distribution ?

$$1 + z = \frac{E_{em}}{E_{rec}}$$



$$\text{with} \quad E = -g_{\alpha\beta}\left(\frac{dx^\alpha}{dp}\right)_{\text{ph}}\left(\frac{dx^\beta}{d\tau}\right)_{\text{obs}} \xrightarrow[\text{source \& obs}]{\text{comobile}} E = \left(\frac{dt}{dp}\right)_{\text{ph}}$$

$$\boxed{\begin{array}{c}\text{geodesics}\\ + \\ \text{isotropy (photon)}\end{array}} \xrightarrow{T = 1/a} \frac{d\left(aE\right)}{dp} = \frac{1}{2}Ka^2\left(\frac{dx^i}{dp}\frac{dx^j}{dp}\right)_{\text{ph}}\frac{\partial\theta_{ij}}{\partial T}$$

Simplest « mono-mode » case

Let us consider the case where the free Fourier amplitudes are chosen as

$$\tilde{C},\overline{C},\tilde{S},\overline{S},\tilde{V},\overline{V},\tilde{\Psi}_i,\overline{\Psi}_i \propto \delta\left(\vec{u}-\vec{\sigma}\right)$$

general case = superposition of such mono-mode cases

Just one mono-mode : choose *xyz* in such a way

$$\vec{\sigma} = \delta\begin{pmatrix} 0 \\ 0 \\ \sigma > 0 \end{pmatrix}$$

$$\begin{pmatrix} B_{11} \\ B_{22} \\ B_{33} \\ B_{12} \\ B_{23} \\ B_{13} \end{pmatrix} = \Omega \, . \begin{pmatrix} \sin(2\alpha) \\ -\sin(2\alpha) \\ 0 \\ \cos(2\alpha) \\ 0 \\ 0 \end{pmatrix} \delta\left(\vec{u}-\vec{\sigma}\right) \quad \& \quad \begin{pmatrix} \tilde{V} \\ \overline{V} \\ \tilde{\Psi}_i \\ \overline{\Psi}_i \end{pmatrix} = \begin{pmatrix} \tilde{v} \\ \overline{v} \\ K\tilde{p}_i \\ K\overline{p}_i \end{pmatrix} \delta\left(\vec{u}-\vec{\sigma}\right)$$

This leads to

$$z_{\vec{N}} = z_{dS} - \frac{1}{2}\left(1 + z_{dS}\right)\Delta J \qquad \text{with} \qquad z_{dS} = \frac{a_{obs}}{a} = \text{de Sitter shift}$$

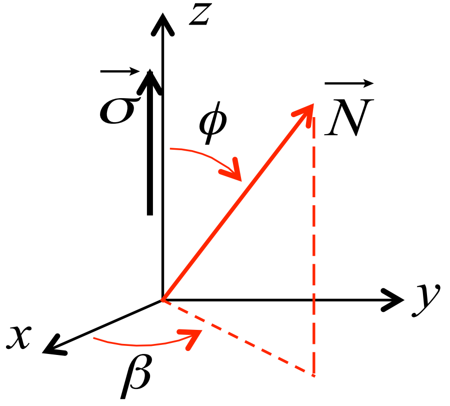
$$
\begin{aligned}
2\Delta J = {} & \underbrace{\left[\tilde{c}\sin(2\beta+2\tilde{\alpha}_C)+\bar{s}\sin(2\beta+2\bar{\alpha}_S)\right]}_{}\underbrace{f\left(\frac{\sigma}{a},\frac{\sigma}{a_{obs}},N^z\right)}_{}+\underbrace{\left[\tilde{c}\sin(2\beta+2\tilde{\alpha}_C)-\bar{s}\sin(2\beta+2\bar{\alpha}_S)\right]}_{}\underbrace{f\left(\frac{\sigma}{a},\frac{\sigma}{a_{obs}},-N^z\right)}_{} \\
& +\underbrace{\left[\bar{c}\sin(2\beta+2\bar{\alpha}_C)+\tilde{s}\sin(2\beta+2\tilde{\alpha}_S)\right]}_{}\underbrace{g\left(\frac{\sigma}{a},\frac{\sigma}{a_{obs}},-N^z\right)}_{}+\underbrace{\left[\bar{c}\sin(2\beta+2\bar{\alpha}_C)-\tilde{s}\sin(2\beta+2\tilde{\alpha}_S)\right]}_{}\underbrace{g\left(\frac{\sigma}{a},\frac{\sigma}{a_{obs}},N^z\right)}_{} \\
& +\left(\tilde{v}+\frac{\bar{p}_3}{\sigma}\right)\underbrace{F\left(\frac{\sigma}{a},\frac{\sigma}{a_{obs}},N^z\right)}_{}+\left(\bar{v}-\frac{\tilde{p}_3}{\sigma}\right)\underbrace{G\left(\frac{\sigma}{a},\frac{\sigma}{a_{obs}},N^z\right)}_{}
\end{aligned}
$$

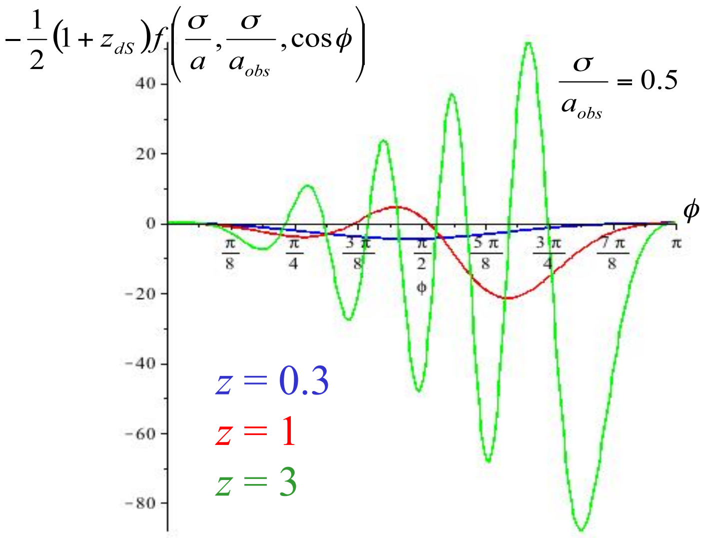
where $\tilde{c},\tilde{\alpha}_C,\ldots,\bar{v},\bar{p}_3 = 12$ csts

- « simple » β-dependence —
- complex ϕ-dependence —

  (encoded in the fonctions f,g,F,G)

$$N^z = \cos\phi$$





Results :

-1- general case OK

-2- C → 0 limit ok : gravitational waves on Minkowski, ….
if C not zero, propagation with variable amplitude

-3- longitudinal vs colatitude dependences (per mono-mode)

-4- aniso contrib → 0 when *z* → 0 (mono-case & general)
not fully obvious (Kasner)

-5- aniso contrib → 0 when *t* → 1 *inside a sphere of given z* (idem)
generalizes what happens in the homog case : isotropization by expansion

-6- amplitude of A*J* increases with *z* (coherence with -4-)

-7- « latitudinal » number of oscillations in A*J* increases with *z*
anisotropy (latitude) angular size decreases with distance

Going further (?)

- what happens/changes if matter (dust) is present ? (realistic cosmo)
- comobility hypothesis → local impact of cosmo. cst. on local motions ?
- link with observed dynamics in clusters ? (In our local group ?)
- ……