Quantum gravity: a phenomenological perspective

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Quantum gravity: a phenomenological perspective

- Naive consequences of QG :
  - Minimal length or ``size'',
  - QFT and its domain of validity,
  - Maximal acceleration & force,
  - Maximal energy density&pressure,
  - Holographic principle,
  - Energy-dependent hbar.
- A tentative geometrization: covariant mass bound conjectures
- Remarks on Minimal Length Scale scenarios, GUP, and vacuum energy

Fundamental constants and their possible meaning

- Intuitively, quantum gravitational world is built on fundamental length, time, and energy scales (of order of the Planck units, say)
- In this point of view, the emergence of c, h, G is non trivial and linked to fundamental kinematics/dynamics of the QG world.

$$c = \frac{l_p}{t_p}, \quad G = \frac{l_p^3}{m_p t_p^2}, \quad \hbar = \frac{m_p l_p^2}{t_p},$$

- Most intuitive example : causality in QG give raise to an universal limiting speed c = lp/tp.
- What are the other physical effects leading to the emergence of G and h ?

Fundamental constants and their possible meaning

- Standard papers : hbar is structural, as is c, because leading to a whole new description of physics ; but G is somehow « only » a coupling constant.
- I actually disagree with this : Schwarzschild result G m/l c^2 < 1/2 is one of the main unexpected result of GR, and its main non trivial consequence, pretty much like the E=mc^2
- G/c^2 is structural constant as well, in the sense that m < l c^2 / 2G
- Non intuitive aspect : not a direct bound on the density, but on the lineic mass which is bounded from above
- On the other hand hbar is less intuitive

Basic inequalities

- Naive QG :consider a system of mass m (or energy E), of ``size'' l, observed during t.
- Then we have
  - $l/t < c$
  - $G\, m/ l\, c^2 < 1/2$
  - $\sigma = m\, l^2/t > \hbar$ (quantum of action)
- Problems :
  - What do we mean by size of a physical system (outside spherical symmetry)?
  - Schwarzschild only apply in sph. symmetry
  - Not very clear what t refers to.

Basic inequalities

- Assume for now

Law 1. *Existence of a maximal speed. Locally, and in any frame, the speed v of physical systems must satisfy:*

$$v \leq c. \tag{2}$$

Law 2. *(First formulation). Existence of a mass bound. In $D = 4$ spacetimes, any physical system with typical size l has a maximal mass m given by:*

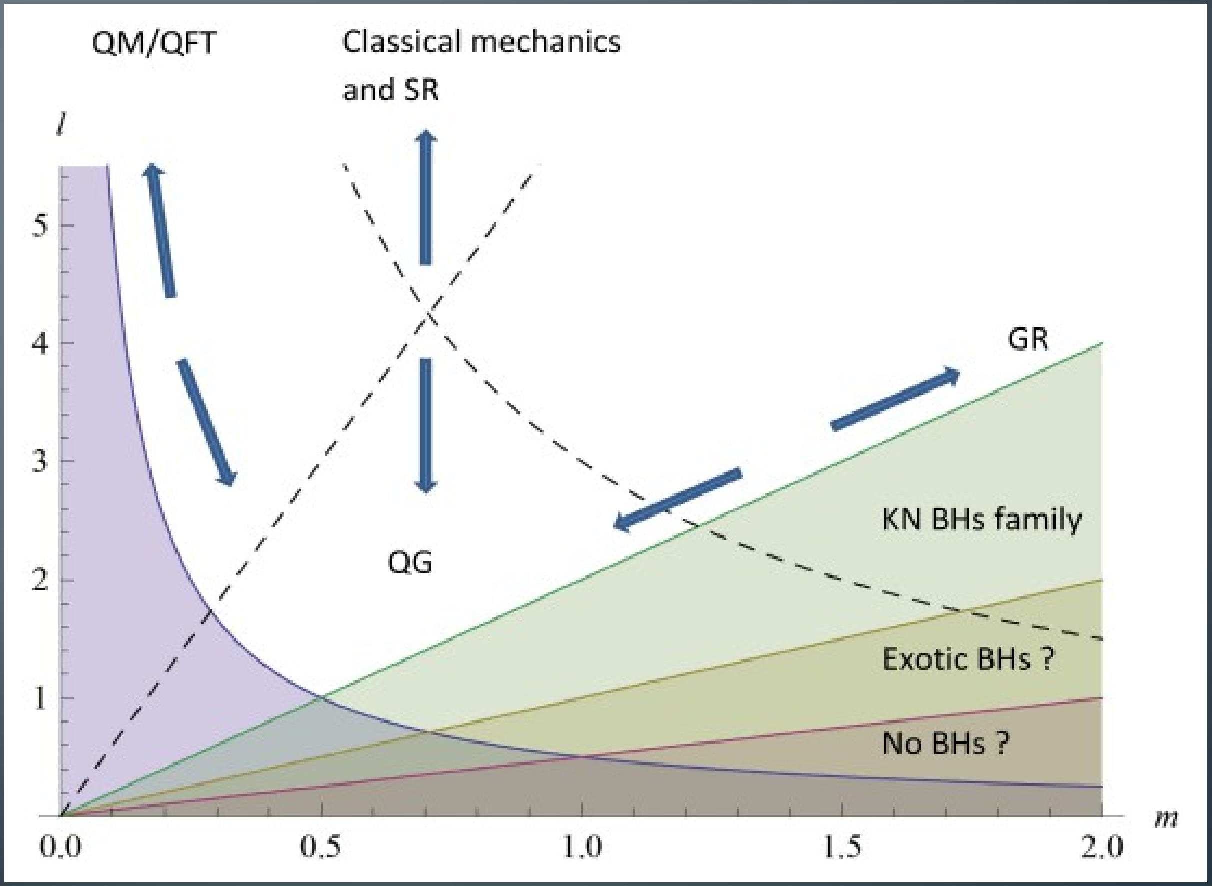
$$\frac{Gm}{lc^2} \leq \mathcal{O}(1). \tag{4}$$

Law 3. *Existence of the quantum of action. There exists a positive pure number β such that any physical system of size l and mass m satisfies*

$$ml \geq \frac{\beta\hbar}{c}. \tag{8}$$

Consequence 1 : minimal "size"

- m l > 1 and m < l and thus l > Max(m,1/m) >1 : minimal ``size" of physical systems or minimal uncertainty Δ x > 1
- But no bounds on the mass m.



… But no minimal length!

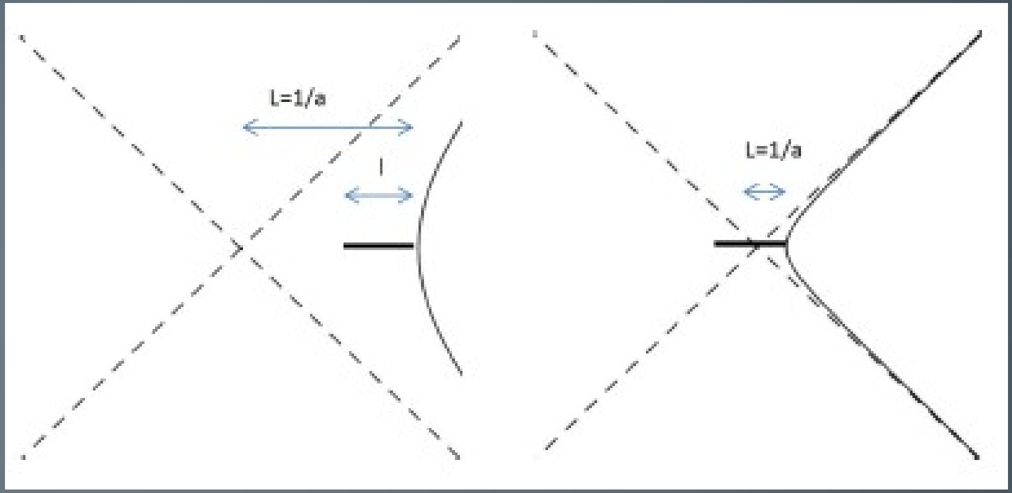
- Consider now a box L filled with one quanta $p=h/\lambda$
- Use $m/L<1/2$ and $m=E/c^2=h/\lambda$
- Get $1/L < \lambda < L$: subplanckian wavelength/transplanckian frequencies allowed (ie without the box collapsing into a BH)
- In strong disagreement with the basic idea of DSR (Doubly Special Relativity with an invariant Planck length)
- No paradox : the box must be larger than Planck length, but can be filled with a subplanckian quanta.
- Physically because the quanta is delocalized, and Einstein eqs. care only about the energy density. Density can be subplanckian while quanta's momenta is transplanckian.

Csq 2: Holographic properties

- Consider a system of mass m made of N dof, with m = N ω
- Use prev. result : $1/l < \lambda < l \Rightarrow 1/l < \omega < l$
- Thus get $$\frac{m}{l} < N < ml$$
- Right hand side similar to Bekenstein bound
- $$N \sim S \leq 2\pi ER$$
- And using m < l, we get the holographic principle in its naive form $$\mathcal{O}(1) < N < l^2$$
- Remark, entropy bound saturated requires m ~ l (hence heavy m>1 systems), but N << l^2 when m << l ; and N ~ 1 iff m ~1/l : systems on Compton line have no substructures.
- (Later generalized to S < A/4, covariant entropy bound, Bousso)

Csq 3: maximal acceleration

- Objects have finite size + special relativity : a < 1/ l



Moreover l>1, thus a<1 bounded by (the huge) Planckian acceleration

Csq 3: maximal acceleration and force

- But there may exists a much tighter bound !

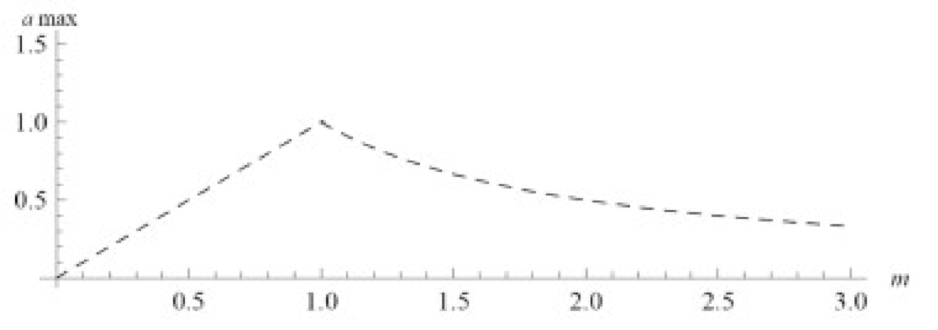
$$\begin{cases} a < \dfrac{1}{l} < m < 1 & \text{if} \quad m < 1 \\ a < \dfrac{1}{l} < \dfrac{1}{m} < 1 & \text{if} \quad m > 1 \end{cases} \qquad (17)$$

- Insert back fundamental constants, and get

$$\begin{cases} a < \dfrac{mc^3}{\hbar} & \text{if} \quad m < 1 \\ a < \dfrac{c^4}{Gm} & \text{if} \quad m > 1 \end{cases}$$

->Caianiello's max acc (1980)

->New ?



Csq 4: maximal force and power

- Similarly, mass-dependent maximal force

$$\begin{cases} f < \dfrac{m^2 c^3}{\hbar} & \text{if} \quad m < 1 \\ f < \dfrac{c^4}{G} & \text{if} \quad m > 1 \end{cases} \qquad (23)$$

- And power, with P_max=f_max c

- For both acceleration, force, and power, the weird thing is that maximal allowed values in the deep m <<1 regime or m>>1 regime are only quantum relativistic (h,c) or respectively general relativistic (G,c) !!
- Needs more thinking and modelling !

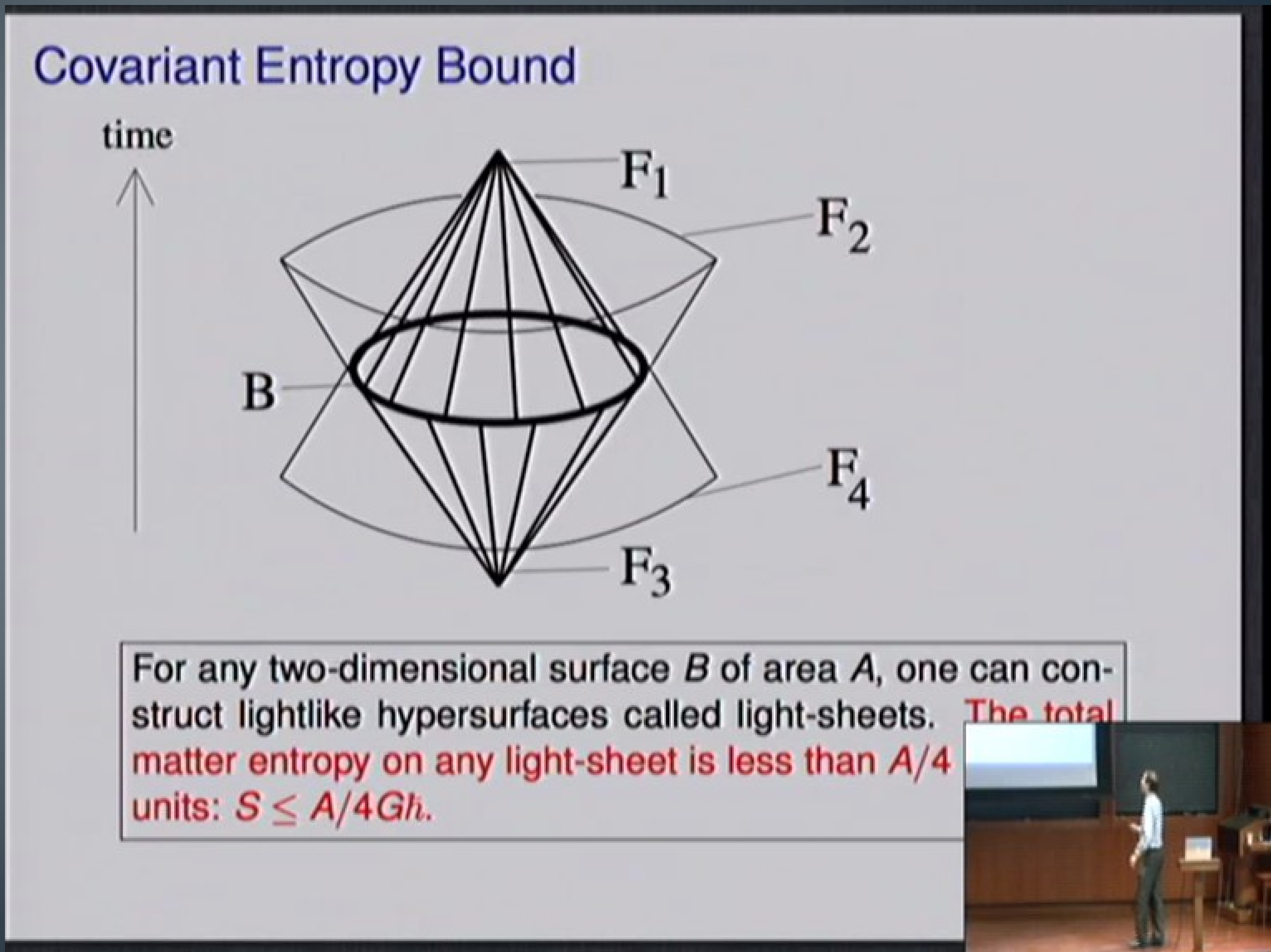
Tentative geometrization

Back to our naive inequalities. What is the size of system ? Example, of a spaghetti ? We need to go beyond Schwarzschild result, ie beyond a spherical symmetric mass bound. Also, how do we get a covariant bound ?

Entropy bound debate (Bekenstein and many others)

->R. Bousso in 98/99 « covariant entropy bound » :

Tentative geometrization



Tentative geometrization

So why not a covariant mass bound ?

Law 2. *(Second formulation). Covariant mass bound conjecture. In a four-dimensional spacetime which is free of horizons, and given any surface B of codimension 2, the integrated mass on the light-sheets $L[B]$ must satisfy:*

$$m[L[B]]^2 < \frac{A[B]}{16\pi G^2}, \tag{5}$$

(Tricky part : requires a local definition of mass)

Plausibility ? If a spacetime has some static event horizons, then its mass satisfies : $16\pi m^2 \geq A_T$

This is the Penrose inequality/conjecture ; and the covariant mass bound is somehow nothing more than its converse

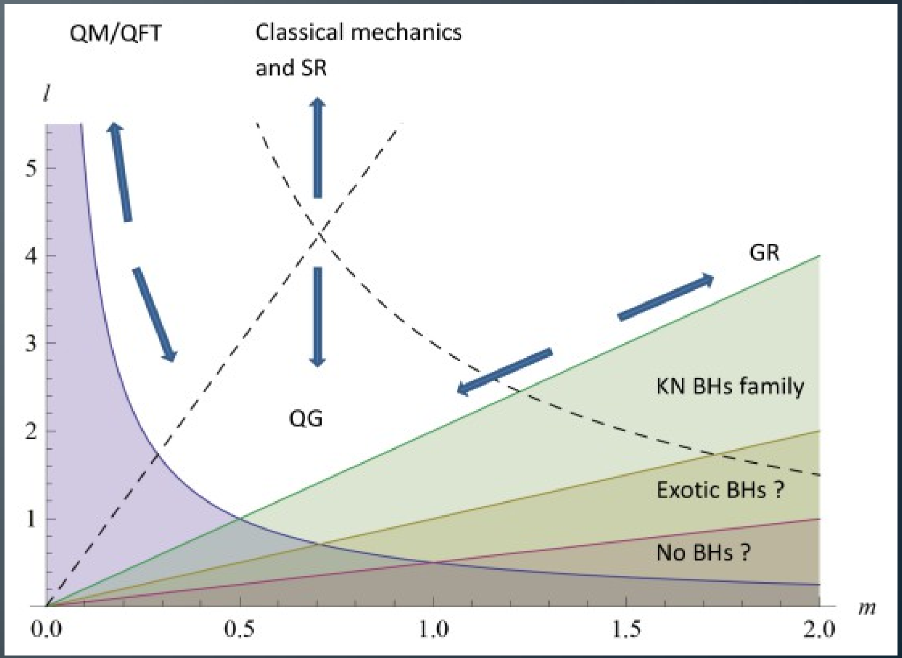
Tentative geometrization

Example : Kerr Newmann Black Hole family has

$$A_H = 4\pi\left(2m^2 - q^2 + 2m\left(m^2 - q^2 - J^2/m^2\right)^{1/2}\right),$$

And thus : $4\pi m^2 \leq A_H \leq 16\pi m^2$.

In terms of l =Sqrt(A/4/Pi),
(areal radius)



And the quantum of action?

In fact squaring the Compton inequality l > hbar/mc, and l->l^2->A

One may write, in 4D, the quantum of action as :

$$A > \frac{4\pi\beta^2\hbar^2}{m^2c^2},$$

(For some pure number beta)

To be compared to the gravitational inequality

$$A > \frac{16\pi G^2 m^2}{c^4}.$$

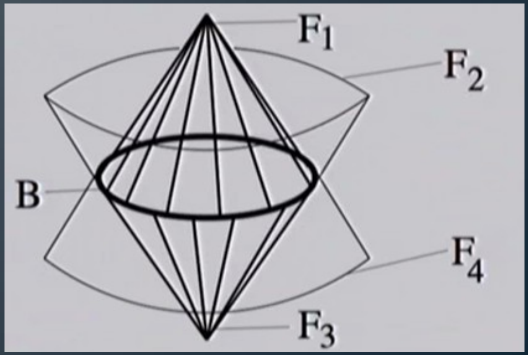
Where one notes the remarkable symmetry

m->1/m

Note : we don't have a minimal size l,

But rather a minimal area in fact

Interpretation ?



Csq: Geometrized version of QG inequalities

Example

Mass/area relations : $4 \pi^2 \hbar^2 / A < m^2 < A/(16 \pi G^2)$

Acceleration : $a<1/l$ may become $a^2 < 1/A$ (Rovelli 2013 in LQG)

Maximal density and pressure $\rho = m/l^3 < 1/l^2$ becomes $\rho < G/A$

etc

Phenomenological models

DSR ? -> maybe not relevant

Minimal uncertainty in position space ? -> Generalized Uncertainty Principle (GUP)

$$\Delta x \Delta p > f(\Delta p^2),$$

where the standard GUP is $f(x) = 1 + \beta x$

Modification of canonical commutation relation -> modif of QM axioms

Max acceleration ? Modification of inertial physics like

$$-d\tau^2 = dx_\mu dx^\mu + \frac{\hbar^2}{m^2c^2} d\dot{x}_\mu d\dot{x}^\mu , ,$$

Non trivial question : effective models of QG, or a new starting point ?

Vacuum energy

Why it shall affect vacuum energy ?

Very roughly (QM) : H = p^2/2m + m w^2 X^2/2

+ Heisenberg principle : Δ x Δp > 1/2 => H ~ p^2 + 1/p^2

Compute its minimum and find H_min = hbar w/2 => vacuum energy

But with the same Hamiltonian and the GUP, it obviously changes

It gives p^2 = mw/Sqrt[4 + b^2 m^2 w^2] -> cste at large w ;

And thus H_min ~ f(w) ~ cst + O(w^2) at large w : even more UV divergent !

But requires a careful identification of H, and a rigorous GUP-QFT calculation

« Conclusions »

Phenomenological approaches to QG in the litterature often based on thought experiments with explicit or implicit use of spherical symmetry (eg. Measurement's precision limited by the formation of a horizon)

However we need to go beyond this symmetry to find more robust mass bounds, and thus « model independent » consequences of QG.

=> Nice covariant mass bounds for both grav. and quantum properties. The area naturally appears as the relevant quantity !

=> Surprising bounds which are not all (h,c,G) dependent, but only (h,c) or (G,c) (though all coming from the fact that minimal size exists)

Model building : what is the point ? Is it only an effective description, or a whole new start ? eg. Max acc and modif of minkowski space