# Quantum vacuum and magnetic fields







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LABORATOIRE NATIONAL DES CHAMPS MAGNETIQUES INTENSES - TOULOUSE

Ce proje a dit coffunce per l'Alon européene. L'Europe trange en MG-Pyrintes avec le fonde surgéen de developpement régional.















### 1. Quantum vacuum seen by an experimentalist in optics

Magnetic and electric properties of a quantum vacuum, R. Battesti and C. Rizzo, Rep. Prog. Phys. **76**, 016401 (2013)

- 2. Inverse Cotton Mouton Effect of vacuum
- 3. Vacuum Magnetic Birefringence

What is «vacuum» mean?

#### \* Aristotle :

\* «The investigation of similar questions about the void, also, must be held to belong to the physicist - namely whether it exists or not, and how it exists or what it is »

\* «A place deprived of body»

\* in Webster's New World Dictionary :

\* 1- a space with nothing at all in it; completely empty space.

\* 2- an enclose space, as that inside a vacuum tube, out of which most of the air or gas has been taken, as by pumping.





Region of space in which a monochromatic electromagnetic plane wave propagates at a velocity that is equal to c.

In classical electrodynamics, vacuum electromagnetic properties are simply represented by two fundamental constants : the vacuum permittivity  $\epsilon_0$  and the vacuum permeability  $\mu_0$ 

Any variation of the velocity of light with respect to c is ascribed to the fact that light is propagating in a medium

$$\mathbf{D}=[\epsilon] \mathbf{E}$$
  
 $n(\mathbf{E},\mathbf{B})=rac{\sqrt{\epsilon\mu}}{\sqrt{\epsilon_0\mu_0}}$   
 $\mathbf{B}=[\mu] \mathbf{H}$ 

$$n \equiv 1$$
$$\frac{\epsilon}{\epsilon_0} = 1 \qquad \frac{\mu}{\mu_0} = 1$$



Vacuum : Lorentz invariant



#### • Only 2 Lorentz invariants in electromagnetism :

$$F = \left(\epsilon_0 E^2 - \frac{B^2}{\mu_0}\right)$$
$$G = \sqrt{\frac{\epsilon_0}{\mu_0}} (\mathbf{E} \cdot \mathbf{B})$$

 $\Box$  Lagrangian has to be relativistic invariant and therefore can only be a function of *F* and *G* 

$$L = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} c_{i,j} F^i G^j.$$



Constitutive equations



$$\mathbf{D} = \frac{\partial L}{\partial \mathbf{E}} \quad \text{et} \quad \mathbf{H} = -\frac{\partial L}{\partial \mathbf{B}}$$
$$\mathbf{D} = 2\epsilon_0 c_{1,0} \mathbf{E} + \sqrt{\frac{\epsilon_0}{\mu_0}} c_{0,1} \mathbf{B} + 2\epsilon_0 c_{1,1} G \mathbf{E} + \sqrt{\frac{\epsilon_0}{\mu_0}} c_{1,1} F \mathbf{B} + 4\epsilon_0 c_{2,0} F \mathbf{E} + 2\sqrt{\frac{\epsilon_0}{\mu_0}} c_{0,2} G \mathbf{E}$$
$$\mathbf{H} = 2c_{1,0} \frac{\mathbf{B}}{\mu_0} - \sqrt{\frac{\epsilon_0}{\mu_0}} c_{0,1} \mathbf{E} + 2c_{1,1} G \frac{\mathbf{B}}{\mu_0} - \sqrt{\frac{\epsilon_0}{\mu_0}} c_{1,1} F \mathbf{E} + 4c_{2,0} F \frac{\mathbf{B}}{\mu_0} - 2\sqrt{\frac{\epsilon_0}{\mu_0}} c_{0,2} G \mathbf{E}$$
with
$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$
$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$
existence of a polarization of a vacuum as in any optical non linear medium

Quantum electrodynamics provides the most complete theoritical treatment for the *c<sub>ij</sub>* coefficient prediction

Quantum Electrodynamics is assumed to be C, P, T invariant

	C	P	T
$ec{E}$	-	-	+
$\vec{B}$	-	+	-
$ec{E}\cdotec{B}$	+	-	-
$E^2 - B^2$	+	+	+

so F is C, P, T invariant but G violates P and T

the only contributions in the lagrangian are the even power of G

$$L = c_{00}F^{0}G^{0} + c_{10}F^{1}G^{0} + c_{20}F^{2}G^{0} + c_{02}F^{0}G^{2} + c_{12}F^{1}G^{2} + \dots$$

Energy density of vacuum

$$U = \mathbf{E}\frac{\partial L}{\partial \mathbf{E}} - L$$

$$U = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) + c_{2,0} \left(\epsilon_0 E^2 - \frac{B^2}{\mu_0}\right) \left(3\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$$

$$+ c_{0,2}\frac{\epsilon_0}{\mu_0} (\mathbf{E} \cdot \mathbf{B})^2 + c_{3,0} \left(\epsilon_0 E^2 - \frac{B^2}{\mu_0}\right)^2 \left(5\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$$

$$+ c_{1,2}\frac{\epsilon_0}{\mu_0} (\mathbf{E} \cdot \mathbf{B})^2 \left(3\epsilon_0 E^2 - \frac{B^2}{\mu_0}\right)$$



3 waves interaction

terms in :



4 waves interaction

terms in :







$$Polarization and magnetization$$

$$D = 2\epsilon_0c_{1,0}\mathbf{E} + \sqrt{\frac{\epsilon_0}{\mu_0}} c_{1,1}\mathbf{B} + 2\epsilon_0c_{1,1}\mathbf{C}\mathbf{E} + \sqrt{\frac{\epsilon_0}{\mu_0}} c_{1,1}\mathbf{F}\mathbf{B} + 4\epsilon_0c_{2,0}F\mathbf{E} + 2\sqrt{\frac{\epsilon_0}{\mu_0}} c_{0,2}C\mathbf{B}$$

$$P = 4c_{2,0}\epsilon_0\mathbf{E}F + 2c_{0,2}\sqrt{\frac{\epsilon_0}{\mu_0}}\mathbf{B}G \qquad \mathbf{M} = -4c_{2,0}\frac{\mathbf{B}}{\mu_0}F + 2c_{0,2}\sqrt{\frac{\epsilon_0}{\mu_0}}\mathbf{E}G$$

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_\omega$$

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_\omega$$

$$P = 4c_{2,0}\epsilon_0(\mathbf{E}_\omega + \mathbf{E}_0)\left(\epsilon_0E_0^2 - \frac{B_0^2}{\mu_0} + 2\epsilon_0\mathbf{E}_\omega \cdot \mathbf{E}_0 - \frac{2\mathbf{B}_\omega \cdot \mathbf{B}_0}{\mu_0}\right) + 2c_{0,2}\frac{\epsilon_0}{\mu_0}(\mathbf{B}_\omega + \mathbf{B}_0)(\mathbf{E}_\omega \cdot \mathbf{B}_0 + \mathbf{E}_0 \cdot \mathbf{B}_\omega + \mathbf{E}_0 \cdot \mathbf{B}_0)$$

$$\mathbf{M} = -4c_{2,0}\frac{(\mathbf{B}_\omega + \mathbf{B}_0)}{\mu_0}\left(\epsilon_0E_0^2 - \frac{B_0^2}{\mu_0} + 2\epsilon_0\mathbf{E}_\omega \cdot \mathbf{E}_0 - \frac{2\mathbf{B}_\omega \cdot \mathbf{B}_0}{\mu_0}\right) + 2c_{0,2}\frac{\epsilon_0}{\mu_0}(\mathbf{E}_\omega + \mathbf{E}_0)(\mathbf{E}_\omega \cdot \mathbf{B}_0 + \mathbf{E}_0 \cdot \mathbf{B}_\omega + \mathbf{E}_0 \cdot \mathbf{B}_0)$$

Heisenberg-Euler Lagrangian

W. Heisenberg and H. Euler, Z. Phys. 98 (1936), 714

Fields can create matter if they are strong enough

If they are not strong enough to create matter, electromagnetic fields polarize the vacuum : virtual possibility of creating matter (electron-position pairs)

$$L_{HE} = \frac{1}{2} \left( \epsilon_0 E^2 - \frac{B^2}{\mu_0} \right) + \alpha \int_0^\infty e^{-\eta} \frac{d\eta}{\eta^3} \times \left\{ i\eta^2 \sqrt{\frac{\epsilon_0}{\mu_0}} (\mathbf{E} \cdot \mathbf{B}) \underbrace{\cos\left(\frac{\eta}{\sqrt{\epsilon_0 E_{cr}}} \sqrt{C}\right) + conj.}_{\cos\left(\frac{\eta}{\sqrt{\epsilon_0 E_{cr}}} \sqrt{C}\right) - conj.} + \epsilon_0 E_{cr}^2 + \frac{\eta^2}{3} (\epsilon_0 E^2 - \frac{B^2}{\mu_0}) \right\}$$
with  $C = \left( \frac{B^2}{\mu_0} - \frac{B^2}{\mu_0} \right) + 2i \frac{\epsilon_0}{\mu_0} (\mathbf{E} \cdot \mathbf{B})$ 

 $\alpha$  : fine structure constant

$$E_{cr} = \frac{m_e^2 c^3}{e\hbar}$$
 : critical electric field ( $\simeq 10^{18} \text{ V/m}$ )

Heisenberg-Euler Lagrangian

At the lowest orders in the fields  $(E \ll E_{cr} \text{ and } B \ll B_{cr})$ :

$$L_{HE} = L_0 + L_{EK}$$

with 
$$L_{EK} = c_{2,0}F^2 + c_{0,2}G^2$$

$$c_{2,0} = \frac{2\alpha^2 \hbar^3}{45m_e^4 c^5} = \frac{\alpha}{90\pi} \frac{1}{\epsilon_0 E_{cr}^2} = \frac{\alpha}{90\pi} \frac{\mu_0}{B_{cr}^2} \simeq 1.67 \times 10^{-30} \left[\frac{m^3}{J}\right],$$
  
$$c_{0,2} = 7c_{2,0}$$

and therefore

$$L_{EK} = \frac{2\alpha^2 \hbar^3}{45m_e^4 c^5} \epsilon_0^2 [(E^2 - c^2 B^2)^2 + 7c^2 (\mathbf{E} \cdot \mathbf{B})^2]$$



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$$\overset{\times}{\longrightarrow}$$

The interaction between two photons of the electromagnetic wave and a photon of the static magnetic field gives rise to a magnetization M.

$$\mathbf{M}_{ICM} = 14c_{2,0}\frac{\epsilon_0}{\mu_0}\mathbf{E}_{\omega}(\mathbf{E}_{\omega}\cdot\mathbf{B}_0) + 8c_{2,0}\mathbf{B}_{\omega}\frac{\mathbf{B}_{\omega}\cdot\mathbf{B}_0}{\mu_0^2}$$

 $\mathbf{E}_{\omega} \| \mathbf{B}_0, \, \mathbf{B}_{\omega} ot \mathbf{B}_0$ 

$$\mathbf{M}_{\mathrm{ICM}\parallel} = 14c_{2,0}\frac{I_{\omega}}{c}\frac{\mathbf{B}_{0}}{\mu_{0}}$$

 $\mathbf{E}_{\omega} \perp \mathbf{B}_{0}, \ \mathbf{B}_{\omega} \| \mathbf{B}_{0}$ 

$$\mathbf{M}_{\mathrm{ICM}\perp} = 8c_{2,0}\frac{I_{\omega}}{c}\frac{\mathbf{B}_{0}}{\mu_{0}}$$

Rizzo et al, EPL, 90, 64003 (2010)

$$c_{2,0} = 1,7 \times 10^{-30} \text{ m}^3/\text{J}$$
  
 $B_0 = 10 \text{ T}$   
 $I_{\omega} = 10^{19} \text{W/m}^2$   
 $\int_{\nabla} \mu_0 \mathbf{M}_{\text{ICM}||} = 8 \times 10^{-18} \text{ T}$ 

#### http://www.izest.polytechnique.edu



ICME in a TGG crystal

A linearly polarized beam induces a magnetization in a medium subjected to a transverse magnetic field







Principle of ICME measurement















Results in a TGG crystal





The signal is proportionnal to the time derivative of the power density of the laser.

A. Baranga et al, EPL 94, 44005 (2011)



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Vacuum...and fluctuations + transverse magnetic field









□ Birefringence and fondamental constants

$$\Delta n_{CM} = \left(\frac{2\alpha^2\hbar^3}{15m_e^4c^5} + \frac{5}{6}\frac{\alpha^3\hbar^3}{\pi m_e^4c^5}\right)\frac{B_0^2}{\mu_0} = \frac{2\alpha^2\hbar^3}{15m_e^4c^5}\left(1 + \frac{25\alpha}{4\pi}\right)\frac{B_0^2}{\mu_0}$$
$$\Delta n_{CM} = k_{CM} \ B^2$$

CODATA 2012

 $k_{CM} = (4.0317 \pm 0.0009) \times 10^{-24} \text{ T}^{-2}$ 

great experimental challenge !

Development of a **very sensitive ellipsometer** in order to be able to observe for the first time this QED prediction



- B at 45° from incident polarization
- Ellipticity to be measured

$$\Psi(t) = \frac{\pi^{\pi}}{\lambda^{\lambda}} \left( \frac{2F}{\pi} \right) B(t)^{2} L_{mag} \sin(2\theta_{p})$$



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Laboratoire National des Champs Magnétiques Intenses



Intense magnetic fields ?

The only method : having a strong current circulating into a coil











Faraday configuration

Temps (ms)

# LNCMI Toulouse 14 MJ, 1 GW



1.0 - 0.8 - 0.6 - 0.4 - 0.2 - 0.1 - 0.0 - 0.1 - 0.0 - 0.1 - 0.1 - 0.0 - 0.1 - 0.1 - 0.0 - 0.0 - 0.1 - 0.0

0.2

► profil :





Fabry Perot cavity



 $L_{\rm c} = 2.27 \, {\rm m}$ 

М,

 $M_1$ 

Ellipticity :  $\Psi(t) = \frac{\pi}{\lambda} k_{CM} \left(\frac{2F}{\pi}\right) B(t)^2 L_{mag} \sin(2\theta_p)$ Finesse :

High reflectivity mirrors  $F \simeq 450\ 000$ 



Fabry Perot cavity



### **D** Photon lifetime $(\tau)$ :







Record :

$$\tau = (1.28 \pm 0.03) \text{ ms}$$
  
 $F = \frac{\pi c \tau}{L} = 530000$ 



### Other cavity around the world :

			<b>J</b> GO	Boold Back Stages of the View
L <sub>c</sub>	3 km	6.4 m	4 km	2.27 m
τ	159 µs	442 µs	970 µs	1.08 ms
$F = \frac{\pi c \tau}{L_c}$	50	70 000	230	450 000
$\Delta v = \frac{c}{2L_{\rm c}F}$	1 kHz	360 Hz	164 Hz	147 Hz



Experimental setup in the cleanroom







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#### Measurements in gases



The experimental points (blue) are fitted with a curve proportionnal to  $B^2$  (red).



Cotton Mouton effect in helium



Smallest effect in nature except for vacuum

Tight test of our apparatus

k<sub>cM</sub> (T `) atm anktb岛ズ3.<sup>2</sup>5 K Very hard experimental task (mirrors rotation, Faraday effect cancellation, systematic effects cancellation)

A. Cadène *et al*, *Phys. Rev. A* **88**, 043815 (2013)





M. Bregant et al., Chem. Phys. Lett. 471, 322 (2009) S. Coriani et al., J. Chem. Phys. 111, 7828 (1999)

Measurements in vacuum

### More than 100 pulses

Systematic effects : take into account the symetry properties of the experiment with respect to the sign of B and the sign of  $\Gamma$ .



A. Cadène et al., arXiv:1302.5389v2 (2013), submitted to PRD



 PVLAS, 2008:
 E. Zavattini et al., Phys. Rev. D 77, 032006 (2008)

 PVLAS, 2012:
 G. Zavattini et al., Int. J. of Mod. Phys. A 27, 1260017 (2012)

 A. Cadène et al., arXiv:1302.5389v2 (2013), submitted to PRD

Best measurement ever done in a vacuum



Conclusions



Development of a very sensitive ellipsometer for mesuring very small birefringences

□ Sophisticated data analysis procedure to overcome systematic effects

□ First measurement of Cotton-Mouton effect of helium compatible with theoritical prediction

Best sensitivity ever obtained for VMB experiments

 $k_{CM} = (5.1 \pm 6.2) \times 10^{-21} \text{ T}^{-2}$ 

A. Cadène et al., arXiv:1302.5389v2 (2013), submitted to PRD



Perspectives



#### □ Magnetic field enhancement : XXL-coils





New set up design for 2014 with an improvement of the whole optical sensitivity

and what about the magnetic birefringence effect in vacuum ?

More and more realistic







## Permanent staff :

Rémy Battesti Mathilde Fouché Carlo Rizzo

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Agnès Souquet

Alexandre Bacou

Paul Berceau (2009-2012) Agathe Cadène (2012-...)

PhD

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